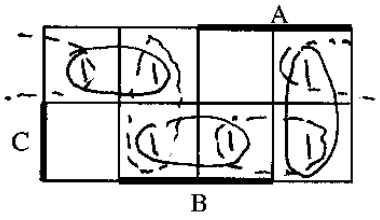
Student Name: _____ SID: _____

Question 1. *Terminology*

(i) *Prime Implicants* (5 Points): In a 3-variable K-map, show a function and its prime implicants that illustrates the fact that there is more than one equally good minimized Sum of Products realizations of the function.

(a) Fill in the K-map for F(A,B,C) below.
(b) Identify the prime implicants by suitable circlings.
(c) Select two different covers from the prime implicants, each with the same number of product terms and literals.

other Solutions possible



3 pts

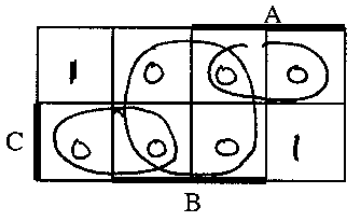
First minimized implementation of F(A,B,C) = $AB' + A'C' + BC$ 1pt

Second minimized implementation of F(A,B,C) = $A'B + AC + B'C'$ 1pt

(ii) *Product of Sums versus Sums of Products* (5 Points): Give a K-map and an example function whose minimized PoS form has fewer literals in it than its minimized SoP form.

(a) Fill in the K-map for F(A,B,C) below.
(b) Indicate the minimized SoP and PoS realizations you obtain.
(c) How many literals is in each realization?

other Solutions possible



Minimized SoP realization of F(A,B,C) = $A'B'C' + AB'C$ 2pts

Number of literals is: 6

Minimized PoS realization of F(A,B,C) = $B'(A'+C')(A+C')$ 3pts

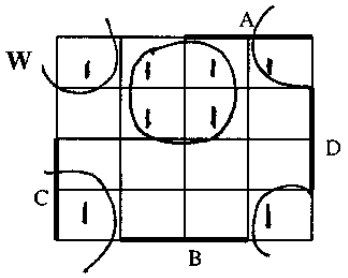
Number of literals is: 5

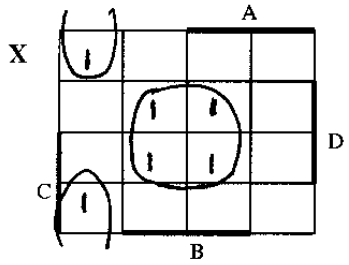
Student Name: _____ SID: _____

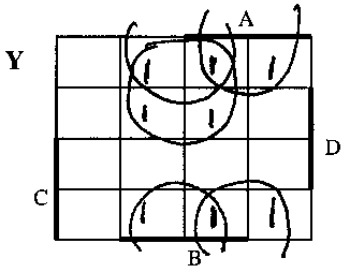
Question 2. *Minimization* (15 Points)
Given the following four four-variable functions:

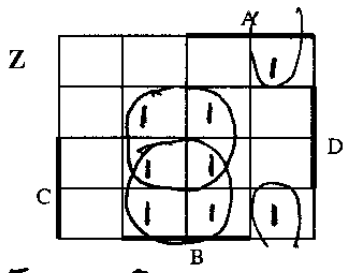
W(A,B,C,D) = Σm(0,2,4,5,8,10,12,13)
X(A,B,C,D) = Σm(0,2,5,7,13,15)
Y(A,B,C,D) = Σm(4,5,6,8,10,12,13,14)
Z(A,B,C,D) = Σm(5,6,7,8,10,13,14,15)

Use K-maps to independently minimize the functions in Sum of Products form.









W = 1 BC' + B'D' 2 X = 5 BD + A'B'D' 6

Y = BC' + 3 BD' + 4 AD' Z = 7 BD + BC + 8 AB'D'

4pts

The number of unique product terms is: 8

Now resolve for W, X, Y, Z when the target of implementation is a Programmable Logic Array (PLA):

W = 1 A'B'D' + 2 AB'D' + 3 BC' 3pts X = 5 BD + A'B'D' 1pt

Y = BC' + ~~AB'D'~~ + AB'D' + 4 BCD' 3pts Z = BD + AB'D' + BCD' 3pts

11pts

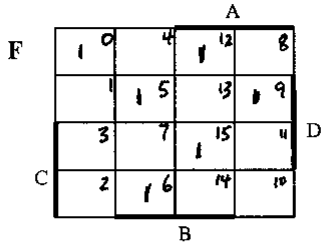
The number of unique product terms is: 5 ← 1 pt

Student Name: _____ SID: _____

Question 3. *Minimization* (20 Points)

Consider the following function: F(A,B,C,D) = Σm(0,5,6,9,12,15)

(a) What is the minimized Sum of Products form? How many literals does it have?

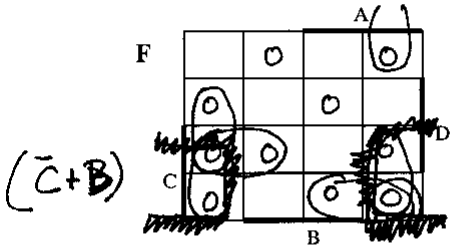


F = $A'B'C'D' + A'BC'D + A'BCD' + AB'C'D + ABC'D' + ABCD$

Literal Count is 24

3pts

(b) What is the minimized Product of Sums form? How many literals does it have?



$(\bar{C}+B)$

F = $(A+B'+C+D)(A'+B'+C+D')(A+B+D')$ ~~$(B+C')$~~ $(A+C'+D')$ ~~$(\quad)$~~ $(A+C'+D)(A'+B+D)$

Literal Count is ~~25~~ 22 3pts

-2 pts if you missed $(B+C')$

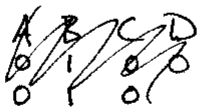
(c) In addition to AND, OR, Complement (NOT) functions, assume that you can also use the XOR function as a primitive gate. They cost no more to use than "simple gates" like AND and OR. How do you obtain an implementation with a minimized literal count now? Write down your answer as a two-level Boolean expression using XOR, AND, OR, NOT functions.

F = $B[A \oplus C \oplus D] + C'[A' \oplus B \oplus D]$ Note: not actually two levels

Literal Count is 8

partial credit based on how close 14pts

Show your work below:



$B(A'C'D + A'CD' + AC'D' + ACD) +$

$B[A'(C'D + CD') + A(C'D' + CD)]$

$B[A \oplus C \oplus D] +$

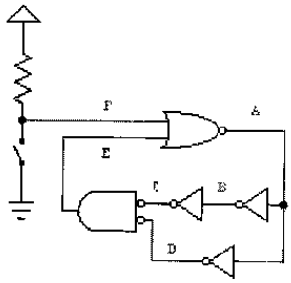
$C'(A'B'D' + A'BD + AB'D + ABD')$

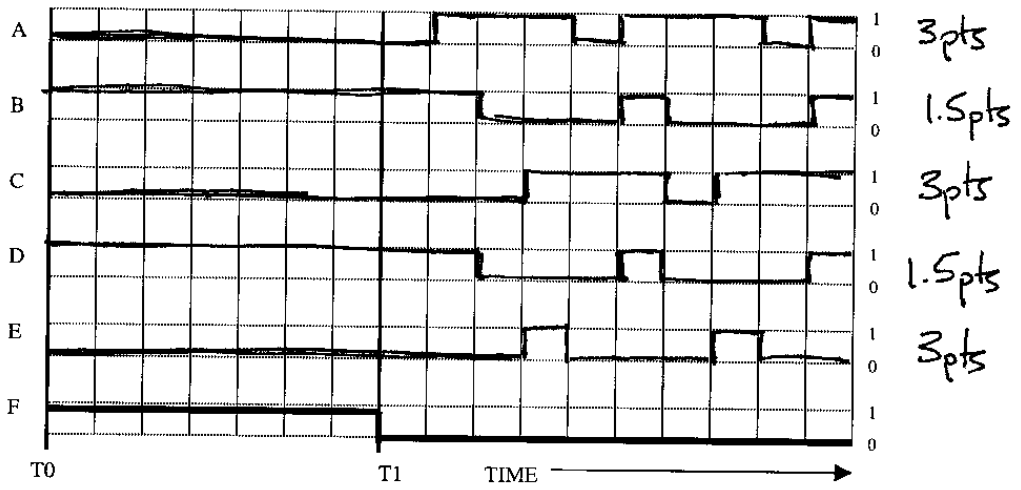
$C'(A'[B'D' + BD] + A[B'D + BD'])$

$C'(A \oplus B \oplus D)$

Student Name: _____ SID: _____

Question 4. *Circuits with Feedback* (15 points)
Consider the following circuit schematic and timing waveform. Assume that all gates have identical gate delays. Each time division in the waveform diagram represents a gate delay. At time T0, the switch has been open long enough for the signals to have reached a steady initial state. At T1, the switch closes. Fill in the waveform diagram.





3pts
1.5pts
3pts
1.5pts
3pts

What kind of periodic signal does this circuit generate? Clock w/ period of 4 gate delays + 75% duty cycle 3pts

Student Name: _____ SID: _____

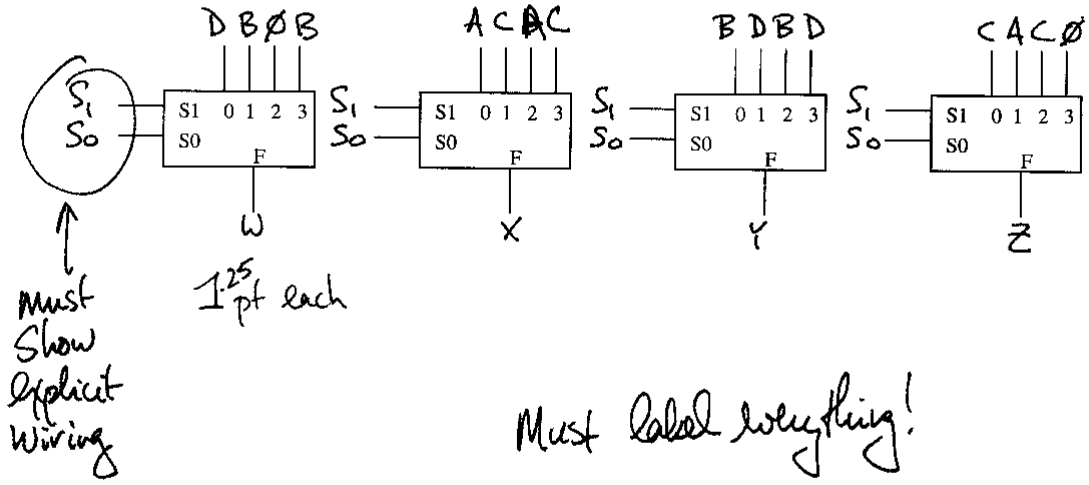
Question 5. *Multiplexer Design* (5 Points)
Design a subsystem using multiplexer components that can do the following function:

Data Inputs: A, B, C, D
Control Inputs: S1, S0
Outputs: W, X, Y, Z

The behavior of the system is described by the following "functional" truth table:

	S1	S0	W	X	Y	Z
Rotate Inputs Right	0	0	D	A	B	C
Rotate Inputs Left	0	1	B	C	D	A
Arithmetic Shift Inputs Right	1	0	0	A	B	C
Arithmetic Shift Inputs Left	1	1	B	C	D	0

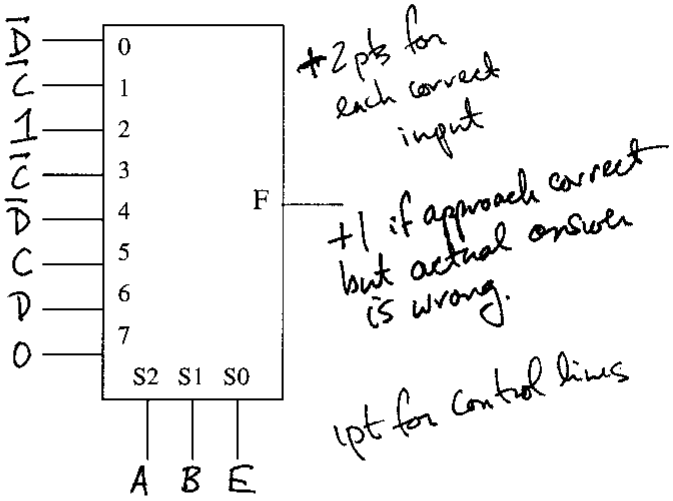
Draw a wiring diagram for your implementation below:



Student Name: _____ SID: _____

Question 6. *Multiplexer Implementation* (15 Points)

Given the five variable function F(A,B,C,D,E) = Σm(0,1,3,4,8,9,10,11,12,14,16,20,21,23,26,30), show how to implement this using a single 8:1 multiplexer and *no other logic gates* (you may assume that variables and their complements are available at no cost to your implementation). HINT: Use A, B, E as the multiplexer control inputs. Show the wiring below:



Show your approach below:

A	B	E	C	D	M	
0	0	0	0	0	0	1
			0	1	2	0 $\overline{D}$
			1	0	4	1
			1	1	6	0
0	0	1	0	0	1	1
			0	1	3	1 $\overline{C}$
			1	0	5	0
			1	1	7	0
0	1	0	0	0	8	1
			0	1	10	1 1
			1	0	12	1
			1	1	14	1
0	1	1	0	0	9	1
			0	1	11	1
			1	0	13	0
			1	1	15	0

A	B	E	C	D	M	
1	0	0	0	0	16	1
			0	1	18	0 $\overline{D}$
			1	0	20	1
			1	1	22	0
1	0	1	0	0	17	0
			0	1	19	0 C
			1	0	21	1
			1	1	23	1
1	1	0	0	0	24	0
			0	1	26	1
			1	0	28	0
			1	1	30	1
1	1	1	0	0	25	0
			0	1	27	0
			1	0	29	0
			1	1	31	0

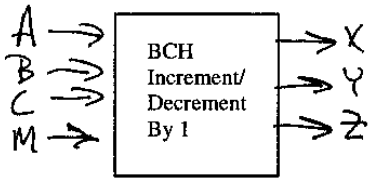
Student Name: _____ SID: _____

Question 7. *Design Problem* (10 Points)
It is well known from the X-Files that Aliens have two arms and three fingers on each hand. So it is not so surprising that they have a base 6 number system.

Design a digital subsystem that takes as input a "binary coded hexary" (BCH) digit (i.e., 0 through 5 is represented by the binary numbers 000 through 101) and a mode input that outputs the BCH digit plus 1 when mode = 0 and BCH – 1 when mode = 1.

(a) Identify your inputs and outputs. Draw a block diagram:

BCH "digit": ABC
Mode : M
BCH output: X Y Z



2pts

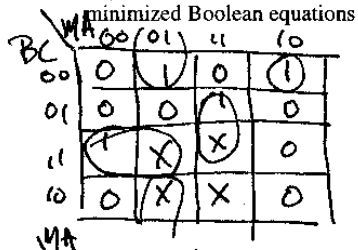
(b) State your assumptions about the behavior of the circuit. Document your understanding of the function with a truth table:

M	A	B	C	X	Y	Z
0	0	0	0	0	0	1
	0	0	1	0	1	0
	0	1	0	0	1	1
	0	1	1	1	0	0
	1	0	0	1	0	1
	1	0	1	0	0	0
	1	1	0	X	X	X
	1	1	1	X	X	X

M	A	B	C	X	Y	Z
1	0	0	0	1	0	1
	0	0	1	0	0	0
	0	1	0	0	0	1
	0	1	1	0	1	0
	1	0	0	0	1	1
	1	0	1	1	0	0
	1	1	0	X	X	X
	1	1	1	X	X	X

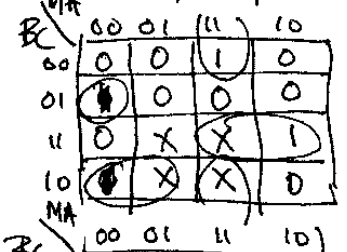
2pts

(c) Implement it in minimized Sum of Products form. Draw filled in K-maps, circled implicants, and the minimized Boolean equations for your outputs:



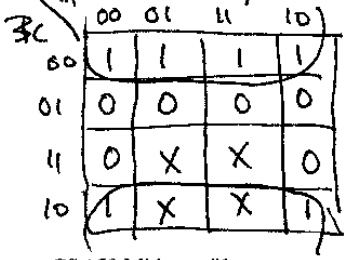
$X = M'BC + M'AC' + MAC + MA'B'C'$

2pts



$Y = M'A'B'C + M'BC' + MAC' + MBC$

2pts



$Z = C'$

2pts