

Final for CS 170

PRINT your name:

(last)

(first)

SIGN your name:

WRITE your section number (e.g., 101):

WRITE your SID:

Three pages of notes is permitted. No electronic devices, e.g. cell phones and calculators, are permitted. Do all your work on the pages of this examination. If you need more space, you may use the reverse side of the page, but try to use the reverse of the same page where the problem is stated.

You have 180 minutes. The questions are of varying difficulty, so avoid spending too long on any one question.

In all algorithm design problems, you may use high-level pseudocode or sketch it in English as long as you make yourself clear.

WARNING: Almost all the answers should be short. No more than a two or three sentences (or a few equations) should suffice. No novels please. Writing more may negatively impact your grade.

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

Problem	Score/Points
Name/Section/etc.	
1	/20
2	/20
3	/20
4	/20
5	/20
6	/20
7	/20
8	/20
9	/20
10	/20
Total	/200

1. Chapter 1.

1. Prove that if $a = x \pmod n$ and $b = y \pmod n$ that $a + b = x + y \pmod n$.
2. Euclid's extended theorem states that for any a, b , there are integers x and y where $\gcd(a, b) = xa + by$. Use this theorem to prove that a has an multiplicative inverse $\pmod p$ if p is a prime.
3. Give an RSA private/public key pair using $p = 5$ and $q = 7$.

2. Chapter2.

1. Give an asymptotic upper bound for the recurrence: $T(n) = 3T(n/2) + O(n^2)$?
2. Give an asymptotic upper bound for the recurrence: $T(n) = T(n/3) + O(n)$.
3. Give an asymptotic upper bound for the recurrence: $T(n) = 2T(\sqrt{n}) + O(\log n)$.
4. What is the basic 8th root of unity? (That is, $\omega^i \neq 1$ for $i < 8$, and $\omega^8 = 1$.)
5. What is the recurrence for the running time for the Fast Fourier Transform on a_0, \dots, a_{n-1} .
6. What is the running time for multiplying two degree $n - 1$ polynomials? Briefly, how?

3. Chapter 3.

1. Consider a depth first search the computed pre and post ordering numberings. Which of the following cannot be the post order numberings and a node u where $pre(u) = 3$ and $post(u) = 13$. If there is an edge (u, v) , which of the following $[pre, post]$ numberings for v is *not* possible: $[5, 8]$ or $[6, 17]$ or $[1, 6]$ or $[2, 14]$ or $[18, 27]$.
2. For a dag, which of the above pairs is not possible?

4. Chapter 4

1. In the union-find data structure with union by rank, give a lower bound on the number of nodes in a rank r tree. (Assume that initially the rank of a node is set to 0.)
2. If in some step of the greedy set cover algorithm one covered 15 out of the 78 remaining elements, give a lower bound on the number of sets required by any optimal solution to the original set cover problem require.

5. Chapter 5

1. What is the running time for decrease-key in a d -ary heap?
2. Give an algorithm for finding the best s - t path where best is defined as the product of the weights on the edges. (You do not need to specify the details of the algorithm if you use a reduction to a problem in the book.)

6. Chapter 6

1. Describe the subproblem definition and update rule for a dynamic programming solution for the Knapsack problem given items $(w_1, v_1) \dots (w_n, v_n)$, that runs in time $O(nV)$, where V is the total value of the items. (In the book, the runtime was (nW) , so be careful. You will get zero credit.)
2. Describe the subproblem definition and update rule for a dynamic programming solution for the Knapsack problem above that does not allow one to reuse items, again your runtime is allowed to run in time depending on V (but not W .)
3. What is the subproblem definition and update rule for a dynamic programming solution for the problem of finding a solving the edit distance problem on strings $x_1 \dots x_n$ and $y_1 \dots y_m$ where the cost of the first gap in a sequence is 2 and a subsequent gap is cost 1. (E.g., the cost of the alignment: $a - -b$ and $axyb$ is 3.)

7. Chapter 7

I want to make shoes: Air Jordans and Ground Raos. I make a profit of 100 dollars on Air Jordans, and a loss of 10 dollars on Ground Raos. To keep myself happy, I have to make at no fewer than 1 Ground Rao for each Air Jordan. I can only make 110 shoes in all, and each Air Jordan takes 2 units of talent where as a ground Rao takes (wastes?) 1 unit of talent. The total talent is 100.

1. Express my profit maximization problem as a linear program.
2. What is the dual program?
3. Give the coordinates of two vertices of the simplex.
4. Illustrate the simplex method on this simplex, beginning from the non-optimal vertex $(0, 0)$.

8. Chapter 7: Flow.

1. Find the residual capacities, $r(e)$, and a residual path in the following partial solution to the following maximum flow problem: It consists of four nodes: s, a, b, c , with arcs: $e_1 = (s, a)$, $e_2 = (s, b)$, $e_3 = (a, t)$, $e_4 = (b, t)$, and $e_5 = (a, b)$. The capacities are $c(e_1) = 4$, $c(e_2) = 5$, $c(e_3) = 3$, $c(e_4) = 3$, $c(e_5) = 4$. The current values of flow is $f(e_1) = 4$, $f(e_2) = 0$, $f(e_3) = 1$, $f(e_4) = 3$, $f(e_5) = 3$.
2. Identify the min cut in the network above.

9. Chapter 8

1. Give a reduction from $3SAT$ (the set of satisfiable formulas with at most three literals per clause) to $3SAT_3$ (the set of satisfiable 3 – SAT formulas where each variable appears at most three times.)
2. Give a reduction from Subset-Sum (given a_1, \dots, a_n and a number K find a subset of the a_i 's that sum to K) to the Knapsack without repetition problem (given a set of items, $(w_1, v_1), \dots, (w_n, v_n)$ and a weight W and a value V find a subset of items that have total weight at most W and value at least V .)

10. Chapter 9

Give a factor of two approximation algorithm for the 2-TSP problem where one is given a set of cities, distances among the cities that obey triangle inequalities, and two cities a and b , where salesmen are based. One wishes to find a tour for each salesman where each city is visited by one of the salesman (each tour return the salesman home.)