Quiz 2

Instructions:
Please write your name at the top of this sheet.
There are two questions on this quiz. Please answer each question on its own sheet of paper, using the back of the sheet to continue your answer. You may assume without proof any result that was proven in class or on a homework, provided you state it clearly.

Please write clear, concise answers. If you are having problems with a part of a question, leave it and try the next one. The two questions carry approximately equal credit.

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1. Consider the following problem:
INSTANCE: A binary Turing machine M.
QUESTION: Does M accept at least 10 strings>

The above problem can be formulated as the problem of recognizing the language
$L_{>=10}$ = { <M> : M accepts at least 10 strings }.

(Hear <M> denotes the standard encoding of a binary TM.)

(a) Show that the language $L_{>=10}$ is recursively enumerable (r.e.).

(b) Recall that the language L_{halt} , defined by
L_{halt} = { <M>$x : M halts on x }

is not recursive. By giveng a reduction from $L_{halt}$to $L_{>=10}$, prove that is $L_{>=10}$ not recursive.
NOTE: You need not show in detail that your reduction can be performed by a TM, but you should show clearly that is maps 'yes'-instances to 'yes'-instances and 'no'-instances to 'no'-instances.

(c) Is the language
$L_{<10}$ = { <M> : M accepts fewer than 10 sttrings }

r.e.? Justify your answer carefully.

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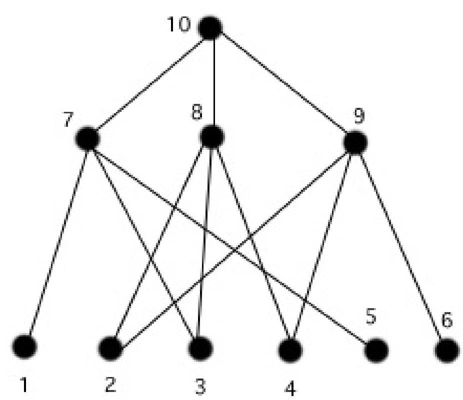
2. The *Steiner Tree* problem, ST is defined as follows:
INSTANCE: An undirected graph G = (V, E) is a subset R _(_ V, and a positive integer k.
QUESTION: Is there a subtree of G that includes all vertices of R and contains at most k edges?

(This problem arises, for example, when it is required to construct a network linking some collection R of sites, using some small number k of existing links (from the set E) and perhaps some additional sites from V.)

(a) Consider the following graph G:



with R = { 1, 2, 3, 4, 5, 6, 10 } and k = 8. Show that this is a 'yes' instance o ST.

(b) Explain briefly why ST belongs to NP.

(c) Prove that ST is MP - complete.

HINT: Try a reduction from the 3-Dimensional Matching Problem, 3DM. The above example should help you.

(d). Does ST remain NP-complete if we restrict attention to instances in which R = V (i.e., *all* sites are to be connected)? Justify your answer carefully.