CS 172, Fall 1999
Midterm
Professor M. Jordan

Problem #1

Closure properties of languages associated with Turing machines.
(a) Prove that the union of two Turing-recognizable languages is Turing recognizable.
(b) Prove that the union of two decidable languages is decidable.

Problem #2

Show that the class of context-free languages is closed under *. (*Hint*: It is probably easiest to do this with a grammer, where a fairly simple construction suffices, but it is doable with PDA's if you prefer).

Problem #3

(a) Draw an NFA that recognizes the languages:
A = {w element of {a,b,c}* | w contains at least two b's or at least one c}.
(b) Give a regular expression that describes this language.

Problem #4

Design a PDA for
L = {(0^i)(1^j) | i does not equal j and i, j >= 0}.
A high-level English description will get yo partial credit, and a diagram will receive full credit.

Problem #5

Let sigma = {0,1,...,9}. Let
L = { | M is a DFA and M does not accept any string containing 555 as a substring}.
Show that L is decidable. (*Hint*: Use the fact that it is possible to contruct a DFA that recognizes the regular language sigma*555sigma*. Also use the fact that regular languages are closed under intersection.).

Problem #6

Let A and B be Turing-reconizable languages. Let (A intersection B) and (A union B) be decidable. Show that A and B are decidable. (*Hint*: Use a Venn diagram and analyze the decidability of various regions of the diagram).

Problem #7

Consider the problem of testing whether a two-tape Turing machine ever writes a nonblank symbol on its second tape. Formulate this problem as a language. Show that this language is undecidable. (*Hint*: Use a reduction from (A)tm. The basic idea is to contruct a two-tape machine that simulates a Turing machine M on string w.).