Final Exam

Name:

This test has **seven** questions and eight numbered pages. Answer on the question paper. You may write on the back of the paper.

You may use any result that was stated or proved in lectures without giving the proof, as long as you state the result clearly. (It is not necessary to state the theorem number.) For full marks, you must justify your answers.

Name:

Question 1 (10 marks)

Let L be the language of strings over the alphabet $\{0, 1\}$ that contain the substring 010 exactly once. Note that 01010 contains 010 twice. Give a minimal DFA that accepts L. (For full marks, you must prove minimality.)

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Question 2 (15 marks)

Define a *collector automaton* in the same way as a DFA $M = (Q, \Sigma, \delta, q_0, A)$ but with the following definition of acceptance. The collector automaton accepts an input w if, and only if, the sequence of states it visits when reading w includes every accepting state. Prove that every collector automaton accepts a regular language. Is every regular language accepted by some collector automaton? Give a proof or a counterexample.

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Question 3 (10 marks)

Show that a language $L \subseteq \Sigma^*$ is semi-decidable if, and only if, there exists a decidable language L' such that $L = \{x \mid x\#y \in L' \text{ for some } y \in \Sigma^*\}$, where $\#$ is some character not in Σ.

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Question 4 (15 marks)

Let $K(x)$ be the Kolmogorov complexity of a string x and let M be a Turing machine that, for every input x, either writes $K(x)$ to its output tape and halts, or does not halt. Show that M halts for only finitely many inputs.

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Question 5 (15 marks)

A *kernel* of a directed graph $G = (V, E)$ is a set $K \subseteq V$ such that there are no edges within K and, for every vertex $y \in V \setminus K$, there is at least one edge (x, y) with $x \in K$. Let Kernel be the language of strings encoding directed graphs that have a kernel. Show that Kernel is **NP**-complete.

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Question 6 (15 marks)

Let $F = \{\langle M \rangle \mid M \text{ is a DFA and } L(M) \text{ is finite}\}$. Show that $F \in \mathbf{NL}$.

You may assume any reasonable encoding of DFAs as strings. (For example, by treating DFAs as Turing machines that always move the head to the right, and using the standard encoding of Turing machines.) You may assume that checking that a string is a valid encoding of a DFA can be done in **NL**.

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Question 7 (20 marks)

In this question, we consider deterministic multi-tape Turing machines with a read-only input tape. Let M be a TM and let $f\colon \mathbb{N} \to \mathbb{N}$ be a function such that M that halts after at most $f(|x|)$ steps for every input x. Show that if $n = o(f(n))$, there is a TM M' that decides the same language as M and that halts after at most $f(|x|)/2$ steps for sufficiently long inputs x.

Comment on the relevance of this result to the definition of time-bounded complexity classes.