Midterm 1

Name:

This test has **three** questions and four numbered pages. Answer on the question paper.

You may use any result that was proved in lectures without giving the proof, as long as you state the result clearly. (It is not necessary to state the theorem number.) For full marks, you must justify your answers.

Name:

Question 1 (25 marks)

For any $k > 0$, let L_k be the language of binary strings that *do not* contain k consecutive 1s. For each k, define a DFA for L_k and prove that it is minimal.

Name:

Question 2 (25 marks)

Let $L \subseteq \{0,1\}^*$ be an infinite language. Show that L is decidable if, and only if, there is an enumerator for L that outputs the strings in L in lexicographic order.

(Recall that the lexicographic order on strings puts u before v if, and only if, either $|u| < |v|$ or $|u| = |v|$ and u comes alphabetically before v, i.e., there is some $j \leq |u|$ such that $u_i = v_i$ for $1 \leq i < j$ and $u_j < v_j$.)

Name:

Question 3 (50 marks)

Consider an NFA $N = (Q, \Sigma, \Delta, q_0, A)$ that has no ϵ-transitions (i.e., $\Delta(q, \epsilon) = \emptyset$ for every $q \in Q$). Recall that N accepts its input if there is some possible sequence of transitions that leads to an accepting state.

We define a for-all finite automaton ($\forall$FA) identically to an NFA with no ϵ-transitions, except that we say that a $\forall$FA F accepts input w if, and only if, *every* possible sequence of transitions when reading w leads to an accepting state.

a) Show that the language accepted by a $\forall$FA is regular.

b) Is every regular language accepted by some $\forall$FA?