CS-172 Quiz 2 March 12, 1993
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Each of the following questions counts equally. Try to keep your answers succinct.

1. Define p_x to be the parity bit of $x \in \{0,1\}^*$.

$$p_x = \begin{cases} 0 & \text{if } x \text{ has an even number of 1's} \\ 1 & \text{if } x \text{ has an odd number of 1's} \end{cases}$$

   Design a Turing machine, M, with input alphabet $\{0,1\}$ to compute the function $f(x) = xp_x$. For example, $f(011) = 0110$ and $f(010) = 0101$. You'll receive:

   - 2 points: if your machine properly appends the parity bit onto x.
   - 3 points: if, additionally, your machine accepts with the head at the start of the tape. (It should make it's last transition to the first tape location while entering an accept state.)
   - 4 points: if, in addition, your machine uses no extra tape alphabet symbols.

   Specify the entire machine using proper notation. You may **not** use any extentions of the Turing machine model (such as a 2-way infinite tape), nor can the machine recognize start of tape.

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

   You may specify the transition function with either a table or (to make grading easier) a state diagram which marks transition arrows with X/YD, where $\delta(q, X) = (q', Y, D)$ for symbols X and Y and direction D. (My solution used 9 states. Feel free to use a few more, but your solution should be clear.)

2. Let $L = \{\langle M \rangle : |L(M)| = 1\}$. (I.e., M accepts exactly one string.) Consider the following reduction: $f(\langle M, w \rangle) = \langle M' \rangle$, where $M'(x)$ accepts if $x = \epsilon$ or if $M(w)$ accepts. In all other cases, M' rejects. Recall that $L_u = \{\langle M, w \rangle : w \in L(M)\}$.

   (a) (2 points) If M accepts w, what is $L(M')$? How about if M does not accept w?

   (b) (2 points) Does f reduce $L_u \alpha L$, $\overline{L_u} \alpha L$, $L \alpha L_u$, $\overline{L} \alpha L_u$? (It does one of the four.) Explain. (If you're stuck on this part, you may ask for the solution. You'll lose credit for the part.)

   (c) (2 points) Given your answer above, what can we conclude about L?

   (d) (2 points) What needs to be checked to verify that f is, in fact, a reduction.

   (e) (2 points) Prove that f is a reduction.