
Student Name:

Class Account Username:

Instructions: Read them carefully!

The exam begins at 3:10pm and ends at 6:00pm. You must turn your exam in when time is announced or risk not having it accepted.

Make sure you fill in your name and the above information, and that you sign below. Anonymous tests will not be graded.

Write legibly. *If the person grading the test cannot read something, he/she will simply assume that you meant the illegible portion as a note to yourself and they will ignore it. If you lose points because part of your answer could not be read, you will not be given the opportunity to explain what it says.*

Be clear and concise. *The answers to most questions should be short. If you find yourself writing an excessively long response, you may want to think more carefully about the question. Long rambling answers generally get fewer points than short ones do because there are more opportunities to mark something wrong.*

You may use two pages of notes while taking the exam. You may not ask questions of other students, look at another student's exam, use a textbook, use a phone or calculator, or seek any other form of assistance. In summary: do not cheat. Persons caught cheating will be subject to disciplinary action.

Do not ask questions during the exam. *Most questions are unnecessary and they disturb other students. Figuring out what the exam question is asking is part of the test. If you think you have to make some unusual assumption to answer a problem, note what that assumption is on the test.*

I have read these instructions, I understand them, and I will follow them.

Your Signature: _____

Date: _____

Student ID: _____

Total Points: 256 + 10 You Scored: _____ + Extra _____

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1. Please fill in each of the blanks with an appropriate answer. *2 points each blank, 48 total*

The Euler integration scheme tends to cause simulations to “blow up.” The implicit version of this scheme, known as _____, is much more stable but has a tendency to damp motions artificially.

_____ integration schemes make use of the accelerations at the end of each simulation timestep.

_____ motion capture systems report both the position and orientation of the marker/sensors attached to the subject.

If the singular value decomposition of a matrix is $A = USV^T$, then the psuedo-inverse of the matrix is given by $A^{-P} =$ _____.

The cross product of the tangent vectors of a parametric surface generally can be used to compute the surface _____ vector.

When representing _____ in 3D using homogenized coordinates, the fourth coordinate (i.e. “w”) will be non-zero.

_____ approximates global illumination by making diffuse shading proportional to the un-occluded area over a surface.

Radiance is measured in units of _____.

Catmull-Clark subdivision surfaces are a generalization of uniform, cubic, tensor-product _____ surfaces.

A B-spline curve is always enclosed by the _____ of its control points.

In Catmull-Clark subdivision, the number of new extraordinary points introduced by the third round of subdivision will be _____.

The _____ in the human eye are used in well lit situations.

The _____ of an orthonormal matrix is equal to its *transpose*.

_____ encode 3D rotations as 4D points on the surface of a hyper-sphere.

The special case of a point light source that is infinitely far away from a scene is termed a _____ light.

An orthographic projection is a special case of perspective projection where the _____ is infinitely far away from the scene.

A texture mapping method called _____ is used to change the shape of an object during shading.

NURBS are non-uniform _____ B-Splines that use homogeneous coordinates for control points.

_____ are the dimensionless units used to measure angles.

Finding the intersection of a ray with a sphere requires solving a _____ equation.

Finding the intersection of a ray with a plane requires solving a _____ of equations.

A rigid body will have a constant rotational _____ unless some external force acts on it.

The dynamic range of the human eye is much _____ than the dynamic range of a typical computer monitor.

If a spring with length l has stiffness coefficient k , then a pair of springs in serial with length $l/2$ should have stiffness _____ if they are to replicate the behavior of the original spring.

2. Answer the following questions with True (T) or False (F)

2 points each, 100 total

- Light transport can be modeled reasonably well using a collection of particles attached by springs.
 - The Jacobian of a valid kinematic system will often be invertible.
 - Shiny metal surfaces typically have bright white specularities.
 - Radiance remains constant along parabolic arcs in free space.
 - The implicit representation of a given geometric entity is unique.
 - The rods in the human eye have a spectral response function that peaks between the long and medium cones.
 - Under linear perspective projection, squares always appear as rectangles unless the projection is degenerate.
 - Under linear perspective projection, triangles always will appear to have at least one angle less than 90 degrees.
 - Under perspective projection, most sets of parallel lines will converge.
 - Quaternions represent rotations as points in 3D space on the surface of a sphere.
 - Any set of six non-intersecting polygons can be sorted in front-to-back order.
 - Ink-based color printers could be designed to use other colors besides cyan, magenta, and yellow.
 - Shining an ultraviolet light on scorpions induces a chemical response that causes them to glow green and become agitated, thus making them likely to swarm and attack you.
 - The force exerted by a spring with zero rest length is given by a function that is linear in terms of the endpoint locations.
 - Cubic B-splines curves will be C^2 across segment boundaries.
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- _____ Light fields are generated by non-normalized reflector maps.
- _____ In a kinematic skeleton, every child body must have exactly one inboard joint.
- _____ A rotation matrix always has determinant of -1 .
- _____ Pasteurized coordinates facilitate representing perspective and translation using matrices.
- _____ Ambient occlusion tends to enhance the appearance of surface detail.
- _____ The sky is blue because water vapor adsorbs red light.
- _____ In some women red-green color blindness is caused by a mutation in the coding for the rods.
- _____ A radiosity solver produces a view-independent solution.
- _____ Springs can be used to implement point-to-point constraints between rigid bodies.
- _____ FoldIt is a crowd-sourced system for solving problems relating to abstract origami.
- _____ EteRNA focuses on the problem of designing RNA molecules.
- _____ The rotational and linear velocities of a rigid body, in the absence of additional constraints, are decoupled from each other.
- _____ In a rectilinear spring mesh, adding diagonal springs will help to limit shearing movement.
- _____ Motion graphs used to animation human figures should never contain cycles.
- _____ Given two recorded human motion sequences that appear natural, motions created by blending them will also appear natural and human-like.
- _____ Non-planar inverse kinematics problems will seldom have simple closed-form solutions.
- _____ Planar inverse kinematics problems will seldom have simple closed-form solutions.
- _____ A ball joint represented with a quaternion has three degrees of freedom.
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- _____ C^1 continuity does not always imply G^1 continuity
- _____ G^1 continuity does not always imply C^1 continuity
- _____ The Bezier basis functions are affine invariant.
- _____ The fully explicit version of Euler's method (a.k.a. forward Euler) is unconditionally stable.
- _____ The human eye is uniformly sensitive to all frequencies of visible light.
- _____ Perspective transformations can distort straight lines into circles.
- _____ Some motion capture systems use magnetic fields to determine the location and orientation of tracker objects.
- _____ Pixel-based image representations have infinite resolution.
- _____ Non-zero winding number and parity testing will produce the same result for a polygon with non-self-intersecting boundary.
- _____ Particle systems simulate objects such as waterfalls by modeling the detailed interactions between individual molecules of water.
- _____ The result of applying subdivision to a cubic curve is two quadratic curves.
- _____ Raytracing can be accelerated using BSP-Trees or K-D Trees.
- _____ The short cones in the human eye only sense blue light.
- _____ In a bounding-box tree, the bounding-box stored at a root node may not encompass the boxes of its children nodes.
- _____ The Hermite basis functions have longitudinal support.
- _____ Bump-mapping will not change an object's silhouette.
- _____ Cubic polynomial basis functions can be used to build perfect circles.
-

3. Imagine that you have a RGB monitor where the wires have been swapped so that the red, green, and blue outputs from the computer have been respectively attached to the blue, red, and green inputs on the monitor. When one attempts to display the following colors, what colors will actually appear on the screen? *8 points*

Red _____

Green _____

Blue _____

Cyan _____

Magenta _____

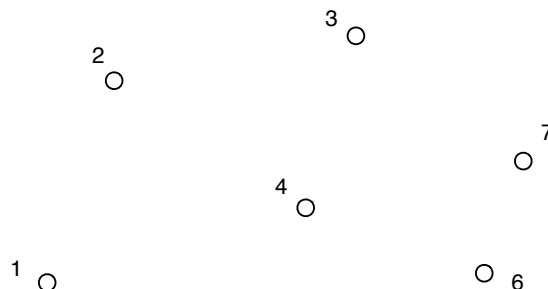
Yellow _____

Black _____

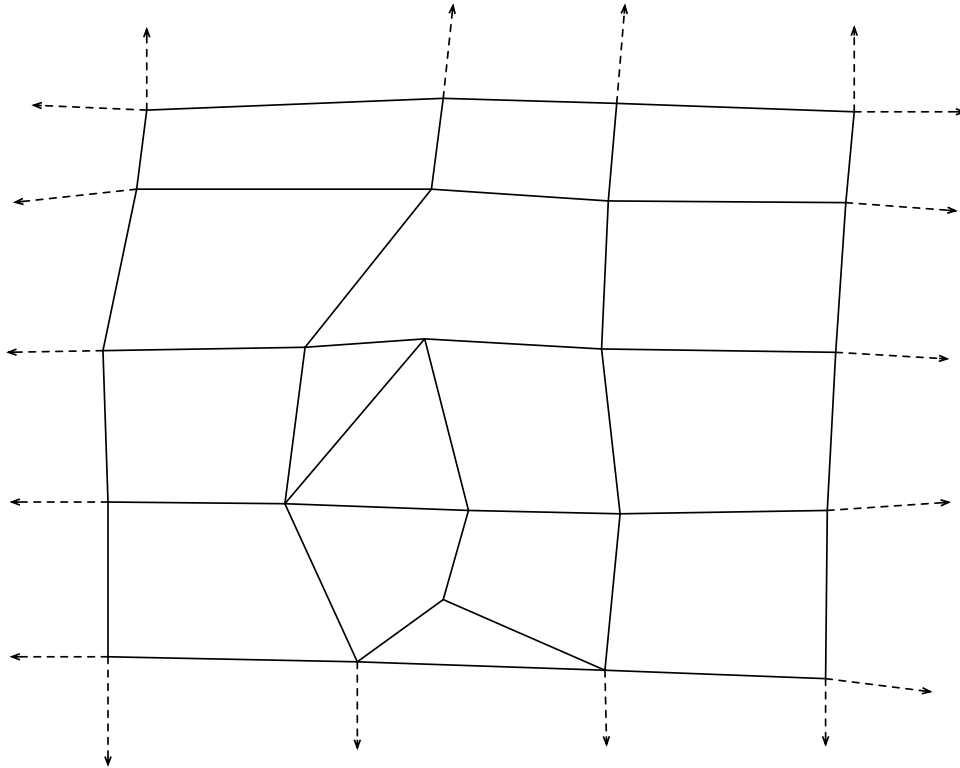
White _____

4. If a surface in 3D is defined parametrically by the function $\mathbf{x} = \mathbf{f}(u, v)$, write out the equation you would use to compute the surface's normal at some point. *4 points*

5. The diagram below shows control points for a curve made by joining two cubic Bezier segments. However control point #5 has been removed. Indicate a location where #5 may be placed to achieve C^1 continuity and draw the curve that would result. Also draw a line where #5 may be placed to achieve G^1 continuity. Make sure your diagram is clear and geometrically reasonable. *9 points*



6. Here is a piece of mesh. Draw the result of applying one iteration of Catmull-Clark subdivision. Then circle all vertices (both original and the new ones you added) that are extraordinary. *Note: I am only interested in the topology of your answer.* 8 points



7. Name a phenomenon that can be modeled easily using photon mapping but that cannot be modeled with a basic ray-tracing algorithm. Give an example. 3 points

8. Briefly describe a useful method for interpolating between two rotations. 4 points

9. Below are two 4x4 homogenized transformation matrices. What does the first one do? How does the effect produced by the second one differ from that produced by the first? *4 points*

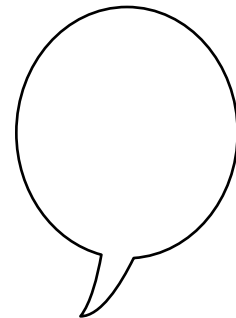
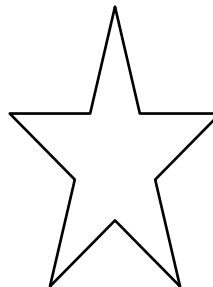
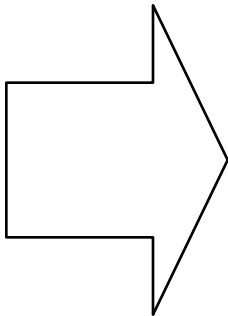
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

The first one will: _____

The second one will: _____

10. Draw the convex hull of the shapes shown below.

6 points



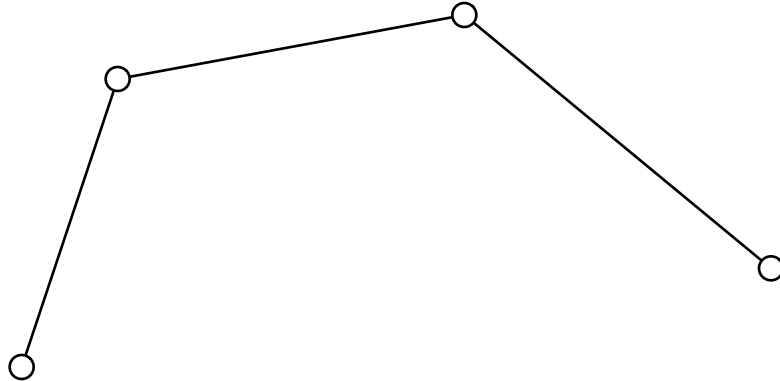
11. Write out an implicit equation for a sphere.

4 points

12. Write out a parametric equation for a line in 3D.

3 points

13. The diagram below is the control polygon for a Bezier curve segment. Draw the curve and show how de Casteljaeu's algorithm can be used to subdivide the curve into two equal halves. Make sure your drawing is geometrically reasonable and shows correct curve tangents for the beginning, middle, and end of each segment. 5 points

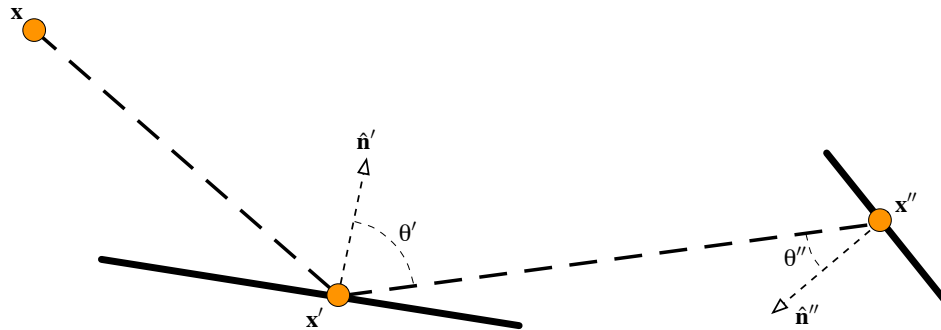


14. In the space below draw a diagram illustrating the formation of a tight *refraction caustic*. Your diagram should be clear and concise, and clearly show the light source(s), the reflecting surface(s), the surface where the caustic forms, and the paths of representative rays. A 2D diagram is suggested. 3 points

15. Given three points, A, B and C, write out a parametric equation in two variables, u and v , for the triangle formed by these three points. 4 points
-

16. Consider the following equation and diagram:

$$L_s(\mathbf{x}, \mathbf{x}') = \delta(\mathbf{x}, \mathbf{x}') \left[E(\mathbf{x}, \mathbf{x}') + \int_S \rho_{x'}(\mathbf{x}, \mathbf{x}'') L_s(\mathbf{x}', \mathbf{x}'') \frac{\cos(\theta') \cos(\theta'')}{\|\mathbf{x}' - \mathbf{x}''\|^2} d\mathbf{x}'' \right]$$



Explain what effects each of the following is responsible for.

10 points

$E(\mathbf{x}, \mathbf{x}')$

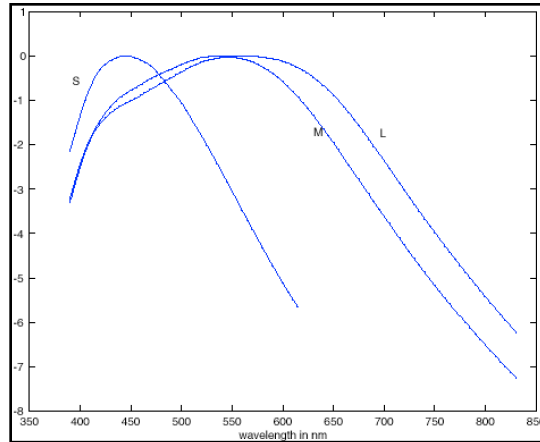
$\delta(\mathbf{x}, \mathbf{x}')$

$\cos(\theta'')$

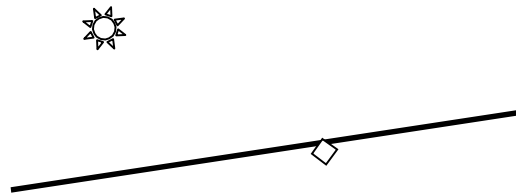
$\|\mathbf{x}' - \mathbf{x}''\|^2$

$\rho_{x'}(\mathbf{x}, \mathbf{x}'')$

17. The following are the response curves for the cones in the human eye. Which type of cone is most sensitive to *red* light? 2 point

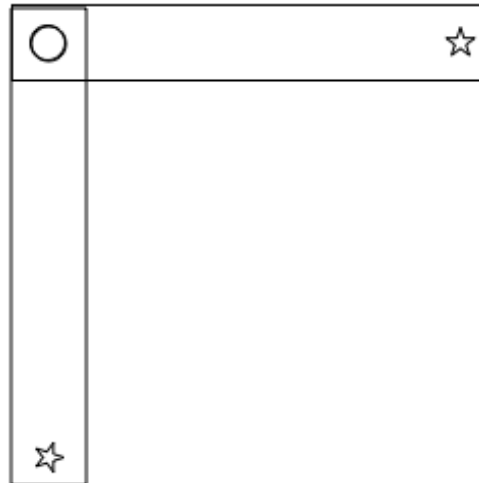


18. Consider the diagram below. A location has been marked on the surface. Indicate a viewer position such that a viewer looking at the surface from that position would see a specular highlight on the surface at the marked location. 3 points

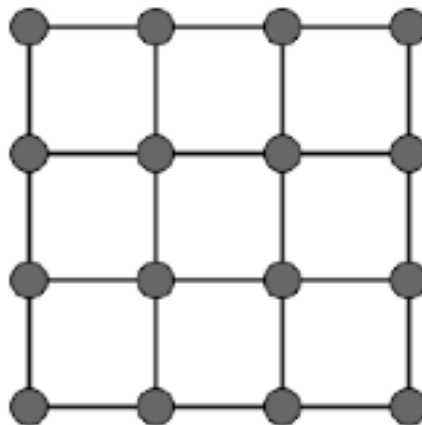


19. When rendering a scene with a ray-tracing method, what part of the solution must be recomputed when the viewer moves? 2 points

20. Below is a diagram showing a bar that has been rotated 90 degrees about the point indicated with a circle. If intermediate positions were generated by linearly interpolating the transformation matrices, how would the point indicated by the star move? Give your answer by drawing the path of the star. 4 points

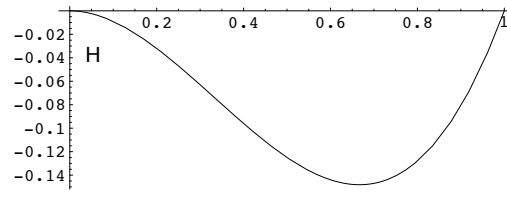
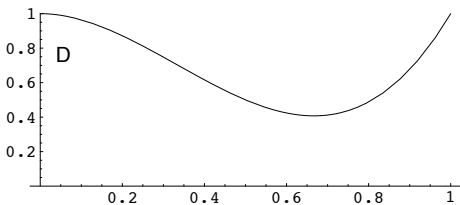
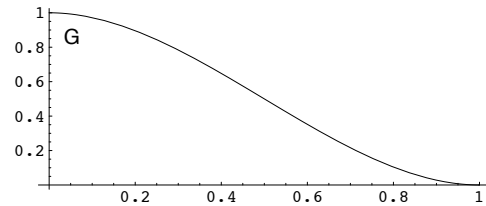
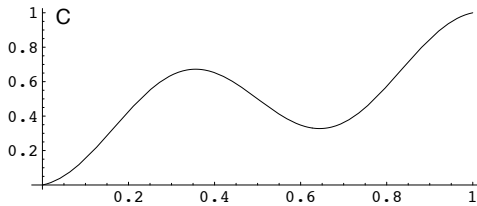
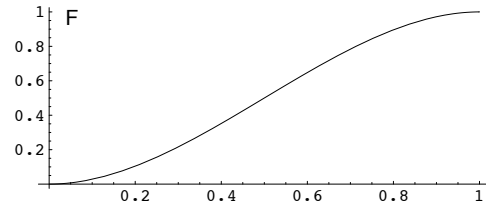
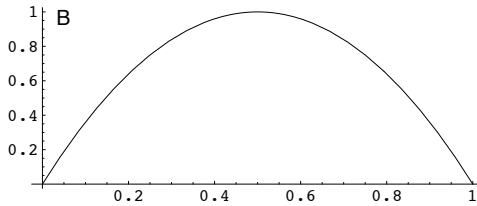
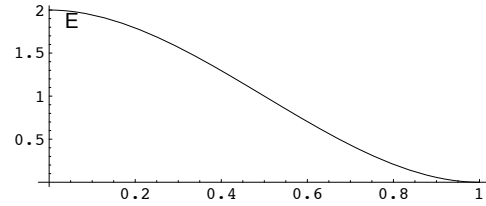
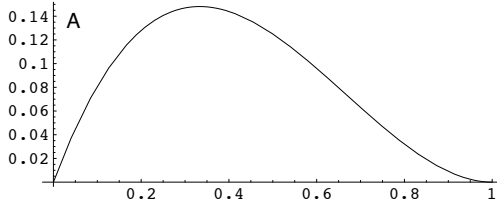


21. On the diagram below, draw the springs that should be added to provide some resistance to in-plane shearing. 4 points



22. I define a curve as $c(u) = \sum_i p_i \phi_i(u)$, $u \in [0..1]$ for some set of basis functions $\phi_i(u)$ and I insist that no matter what values are used for the control points P_i , the resulting curve is always in the convex hull of the control points. What mathematical equation must be true for my assertion to hold? 4 points

23. There are 8 functions plotted below. Neatly cross out the ones that are not part of the cubic Hermite basis set. Next to the remaining plots write what feature of the curve that basis controls. 6 points



For those that are NOT Hermite basis functions write a single short sentence that explains why they could not be. Your reason should be simple. *Note: "It isn't what I have in my notes," "it won't fit," "it doesn't solve the equations," or other generic answers will not be accepted.* 8 points

| Letter | Reason |
|--------|--------|
| _____ | _____ |
| _____ | _____ |
| _____ | _____ |
| _____ | _____ |

24. Would the differences between Phong interpolation (interpolating normals) and Gouraud interpolation (interpolating colors) be more noticeable on a very fine or very coarse mesh?

4 points

EXTRA CREDIT

*10 points***Given a two planes:**

Plane 1: $\mathbf{x}_1(u, v) = \mathbf{a}_1 + u\mathbf{b}_1 + v\mathbf{c}_1$

Plane 2: $\mathbf{x}_2(u, v) = \mathbf{a}_2 + u\mathbf{b}_2 + v\mathbf{c}_2$

Write out an explicit parametric equation that produces the line where the planes intersect. Also indicate when this line is undefined.

Your answer must be neat and clear. No points will be awarded for imprecise or messy answers. Your answer should be in the form of a simple explicit equation that you have drawn a box around. Do not attempt this question until you have completed the rest of the exam! There will be no partial credit for this question.
