CS 188 Final Exam, 5-8 pm, Wed Dec 16, 1998

1. (a)

$$\begin{aligned}\text{Prob}(R|\sim U) &= \frac{\text{Prob}(R\wedge \sim U)}{\text{Prob}(\sim U)}\\ &= \frac{\text{Prob}(R)\text{Prob}(\sim U|R)}{\text{Prob}(R)\text{Prob}(\sim U|R)+\text{Prob}(\sim R)\text{Prob}(\sim U|\sim R)}\\ &= \frac{0.5\cdot 0.3}{0.5\cdot 0.3+0.5\cdot 0.7} = 0.3\end{aligned}$$

(b) It will increase (become greater than 30%) since the umbrella on the 2nd day is an evidence for rain on the 2nd day, which in turn is an evidence for rain on the 1st day since the weather is more likely to stay the same than change.

(c) We'll compute it for the case where $y_1 =\sim U, y_2 = U$. We need to compute α for both values of q_2. Let's simply use the brute force method, by computing the following quantities:

$$\begin{aligned}\text{Prob}(y_1 =\sim U, y_2 = U, q_1 = R, q_2 = R) &= \text{Prob}(R)\text{Prob}(R|R)\text{Prob}(\sim U|R)\text{Prob}(U|R)\\ &= 0.5\cdot 0.8\cdot 0.3\cdot 0.7 = 0.084\\ \text{Prob}(y_1 =\sim U, y_2 = U, q_1 = R, q_2 = S) &= \text{Prob}(R)\text{Prob}(S|R)\text{Prob}(\sim U|R)\text{Prob}(U|S)\\ &= 0.5\cdot 0.2\cdot 0.3\cdot 0.3 = 0.009\\ \text{Prob}(y_1 =\sim U, y_2 = U, q_1 = S, q_2 = R) &= \text{Prob}(S)\text{Prob}(R|S)\text{Prob}(\sim U|S)\text{Prob}(U|R)\\ &= 0.5\cdot 0.2\cdot 0.7\cdot 0.3 = 0.021\\ \text{Prob}(y_1 =\sim U, y_2 = U, q_1 = S, q_2 = S) &= \text{Prob}(S)\text{Prob}(S|S)\text{Prob}(\sim U|S)\text{Prob}(U|S)\\ &= 0.5\cdot 0.8\cdot 0.7\cdot 0.7 = 0.196\end{aligned}$$

Therefore,

$$\begin{aligned}\alpha(q_2 = R) &= \text{Prob}(y_1 =\sim U, y_2 = U, q_2 = R)\\ &= \text{Prob}(y_1 =\sim U, y_2 = U, q_1 = R, q_2 = R)\\ &+ \text{Prob}(y_1 =\sim U, y_2 = U, q_1 = S, q_2 = R) = 0.105\\ \alpha(q_2 = S) &= \text{Prob}(y_1 =\sim U, y_2 = U, q_2 = S)\\ &= \text{Prob}(y_1 =\sim U, y_2 = U, q_1 = R, q_2 = S)\\ &+ \text{Prob}(y_1 =\sim U, y_2 = U, q_1 = S, q_2 = S) = 0.205\end{aligned}$$

(this means that if you see an umbrella on the 2nd day but not the 1st, then it's about twice as likely that it rains on the 2nd day as it is that it doesn't).

2. Let's use the following notation: by $p_k(w)$ we will denote the probability of the most likely sequence of weathers on days $1\ldots k$, such that the weather on the kth day is $w = R$ or S. More specifically,

$$p_k(w) = \max_{q_1\cdots q_{k-1}} P(q_1\ldots q_{k-1}, q_k = w, y_1\ldots y_k)$$

(we will use $y = (\sim U, U, U)$).

Also, by $prev_k(w)$ we will denote the weather on the $(k-1)$th day, in this most likely sequence ($prev_k$ is undefined for $k=1$).

Now we are ready to go:

$$\begin{aligned}
p_1(R) &= 0.5 \cdot 0.3 = 0.15 \\
p_1(S) &= 0.5 \cdot 0.7 = 0.35 \\
p_2(R) &= \mathrm{Prob}(U|R)\max(0.15\mathrm{Prob}(R|R), 0.35\mathrm{Prob}(R|S)) = 0.7\max(0.15 \cdot 0.8, 0.35 \cdot 0.2) \\
&= 0.7 \cdot 0.15 \cdot 0.8 = 0.084 \\
prev_2(R) &= R \text{ (since the first term, corresponding to R, was the largest)} \\
p_2(S) &= \mathrm{Prob}(U|S)\max(0.15\mathrm{Prob}(S|R), 0.35\mathrm{Prob}(S|S)) = 0.3\max(0.15 \cdot 0.2, 0.35 \cdot 0.8) \\
&= 0.3 \cdot 0.35 \cdot 0.8 = 0.084 \\
prev_2(S) &= S \\
p_3(R) &= 0.7\max(0.084 \cdot 0.8, 0.084 \cdot 0.2) = 0.7 \cdot 0.084 \cdot 0.8 = 0.04704 \\
prev_3(R) &= R \\
p_3(S) &= 0.3\max(0.084 \cdot 0.2, 0.084 \cdot 0.8) = 0.3 \cdot 0.084 \cdot 0.8 = 0.02016 \\
prev_3(S) &= S
\end{aligned}$$

Now, $p_3(R) > p_3(S)$, so we know that the 3rd day weather was R, the one before that was $prev_3(R) = R$, and before that — $prev_2(R) = R$.

Answer: (R, R, R) is the most probable sequence.

3. For example, you could use these operators:

   - $Go(x)$ — go to location x
   - $Pick(x)$ — pick up a piece of mail x.
   - $Give(x, y)$ — give this piece of mail to person y, to whom it's addressed.

   We can use feedback control to see what mail has been picked up, thus telling us to what people in what rooms it should be delivered. Then we can use feedback to check whether we could get to the room and into the room, and whether the addressee is there.

4. The threshold h_0 should satisfy this condition:

$$0.7\exp\left(-\frac{(h_0-12)^2}{2 \cdot 3^2}\right) = 0.3\exp\left(-\frac{(h_0-6)^2}{2 \cdot 3^2}\right).$$

   With some algebra, we get

$$\frac{(h_0-6)^2 - (h_0-12)^2}{2 \cdot 3^2} = \ln(0.3/0.7)$$

   or

$$6(h_0 - 9) = 9\ln(0.3/0.7)$$

   which yields $h_0 = 7.73$

5. (a) True. Backpropagation is an optimization method, and the function being minimized is the error on the training set, so it's independent of the test set. We stop the gradient descent when we have reached a minimum of this function.

   Once the optimization has been completed, however, we use the test set to assess the quality of our classifier (which depends on the network structure and the features we use).

   (b) N/A

   (c) False. When the robot starts moving towards a different point, that point will be at the FOE, regardless of speed, so different directions result in different FOEs.

6. Let's use U_1 and U_2 to denote the utilities at the two nonterminal states. Then, we have this system of equations:

$$
\begin{aligned}
U_1 &= -0.4 + 0.6U_2 - 0.1 \\
U_2 &= 0.4U_1 + 0.6 - 0.1
\end{aligned}
$$

   solving which we get $U_1 = -0.2632, U_2 = 0.3947$.