

① a) $g_m = \frac{\partial i_D}{\partial v_{GS}} \Big|_{Q_1} = \frac{C_{ox} W v_{sat}}{2} \left(\frac{1 + \lambda_n v_{DSsat}}{1 + \lambda_n v_{GSsat}} \right) = \frac{(4 \times 10^{-7} \text{ F/cm}^2)(5 \times 10^{-4} \text{ cm})(10^7 \text{ cm/s})}{2} \left(\frac{1 + 0.075}{1 + (0.75)(0.05)} \right)$
 $= 10^{-3} \text{ S} \left[\frac{1.075}{1.0375} \right] \approx 1.04 \text{ mS}$

graphical technique: $g_m = \frac{\Delta i_D}{\Delta v_{GS}} \Big|_{Q_1}$ *typo in scale... should be 0.25 mA*
 $\approx \frac{0.5 \text{ mA}}{0.25 \text{ V}} \approx 2 \text{ mS}$

b) $r_o^{-1} = \frac{\partial i_D}{\partial v_{DS}} \Big|_{Q_1} = \frac{C_{ox} W v_{sat}}{2} (v_{GS} - V_{tn}) \left\{ \frac{\lambda_n}{1 + \lambda_n v_{DSsat}} \right\} = \frac{(4 \times 10^{-7} \text{ F/cm}^2)}{2} \cdot 5 \times 10^{-4} \text{ cm} \cdot 10^7 \text{ cm/s} \left[\frac{0.05}{1.0375} \right]$
 $= 4.82 \times 10^{-5} \text{ S} \Rightarrow r_o = 20.75 \text{ k}\Omega$

c) $g_m = \frac{\partial i_D}{\partial v_{GS}} \Big|_{Q_2} = C_{ox} W v_{sat} \left(\frac{v_{GS}}{v_{DSsat}} \right) \left(1 - \frac{v_{DS}}{2 v_{DSsat}} \right) = 2 \times 10^{-3} \left(\frac{0.25}{0.75} \right) \left(1 - \frac{0.25}{2(0.75)} \right) \text{ S} = 0.5 \text{ mS}$
 graphical technique: $g_m = \frac{\Delta i_D}{\Delta v_{GS}} \Big|_{Q_2} = \frac{0.6 - 0.45 \text{ mA}}{1.75 - 1.5 \text{ V}} = 0.6 \text{ mS}$

d) $r_o^{-1} = \frac{\partial i_D}{\partial v_{DS}} \Big|_{Q_2} = C_{ox} W v_{sat} (v_{GS} - V_{tn}) \left[\frac{1}{v_{DSsat}} - \frac{v_{DS}}{v_{DSsat}^2} \right]; v_{GS} = 1.5 \text{ V}, v_{DS} = 0.25 \text{ V}$
 $= 2 \times 10^{-3} (0.5) \left[\frac{1}{0.75} - \frac{0.25}{(0.75)^2} \right] = 8.9 \times 10^{-4} \text{ S} \Rightarrow r_o = 1.1 \text{ k}\Omega$

$r_o^{-1} \approx \frac{\Delta i_D}{\Delta v_{DS}} \Big|_{Q_2} \approx \frac{0.2 \text{ mA}}{0.25 \text{ V}} \Rightarrow r_o = 1.25 \text{ k}\Omega$


② a) $V_{out} = 2.5 \text{ V} \Rightarrow V_B = V_{out} + V_{BE} = 3.2 \text{ V}$

b) $V_{out} = 2.5 \text{ V}, R_E = 5 \text{ k}\Omega \Rightarrow -I_E = \frac{2.5 \text{ V}}{5 \text{ k}\Omega} = 500 \mu\text{A}$ $I_C = -\alpha I_E \approx 495 \mu\text{A}$

c) common-collector amplifier $\Rightarrow R_{in} = r_{\pi} + \beta_0 (r_o \parallel R_E \parallel R_L)$ $r_{\pi} = \frac{\beta_0}{g_m} = \frac{100(25 \text{ mV})}{500 \mu\text{A}} = 5 \text{ k}\Omega$
 $R_{in} = 5 \text{ k}\Omega + 100(100 \text{ k}\Omega \parallel 5 \text{ k}\Omega \parallel 2.5 \text{ k}\Omega)$ $r_o = \frac{V_A}{I_C} = \frac{50 \text{ V}}{0.5 \text{ mA}} = 100 \text{ k}\Omega$
 $= 169 \text{ k}\Omega$

d) $R_{out} = g_m^{-1} + R_S / \beta_0 = \frac{25 \text{ mV}}{0.5 \text{ mA}} + \frac{5 \text{ k}\Omega}{100} = 50 \Omega + 50 \Omega = 100 \Omega$

e) $A_v = 1$ (exact expression $\frac{1}{1 + \frac{r_{\pi}}{(r_o \parallel R_E)(\beta_0 + 1)}} = \frac{1}{1 + \frac{5 \text{ k}\Omega}{(100 \parallel 5)(101)}} = 0.99$)

f)  $v_{out}/v_s = \left(\frac{R_{in}}{R_S + R_{in}} \right) \left(\frac{R_L}{R_{out} + R_L} \right) = \left(\frac{169}{5 + 169} \right) \left(\frac{2.5}{100 + 2.5} \right) = 0.93$

g) $v_{out}|_{max} = 5 - V_{CEsat} = 4.9 \text{ V}$. $v_{out}|_{min} = 0 \text{ V}$ [but with $I_C \downarrow$, parameters will change hand limit. $\therefore \hat{v}_o|_{max} = v(4.9 - 2.5)/0.93 = 2.55 \text{ V}$

③ a) $I_C = \frac{q D_n A E}{W_B} \cdot n_{pB}(0)$ $n_{pB}(0) = \frac{I_C W_B}{q D_n A E} = \frac{2 \times 10^{-5} \text{ A} \cdot 1 \times 10^{-5} \text{ cm}}{(1.6 \times 10^{-19} \text{ C}) (2.0 \text{ cm}^2/\text{s}) (25 \times 10^{-8} \text{ cm}^2)} = 2.5 \times 10^{14} \text{ cm}^{-3}$

b) $n_{pB}(0) = n_{pB0} e^{V_{BE}/V_{th}} \Rightarrow V_{BE} = V_{th} \ln [n_{pB}(0)/n_{pB0}] = 25 \text{ mV} \ln [2.5 \times 10^{14} / 10^{10}]$
 $n_{pB0} = n_{pB}(0) e^{-V_{BE}/V_{th}} = 2.5 \times 10^{14} \text{ cm}^{-3} e^{-0.925/25} = 233$
 $= n_i^2 / N_A \Rightarrow N_A = \frac{10^{20}}{233} = 4.3 \times 10^{17} \text{ cm}^{-3}$

c) $I_B = \frac{q D_p A E}{W_E} p_{nE}(-x_{BE}) = \frac{1.6 \times 10^{-19} \cdot 5 \cdot 25 \times 10^{-8} \cdot (0.05) (2.5 \times 10^{14})}{7 \times 10^{-6}} = 357 \text{ nA}$

$\beta_F = I_C / I_B \approx 56$