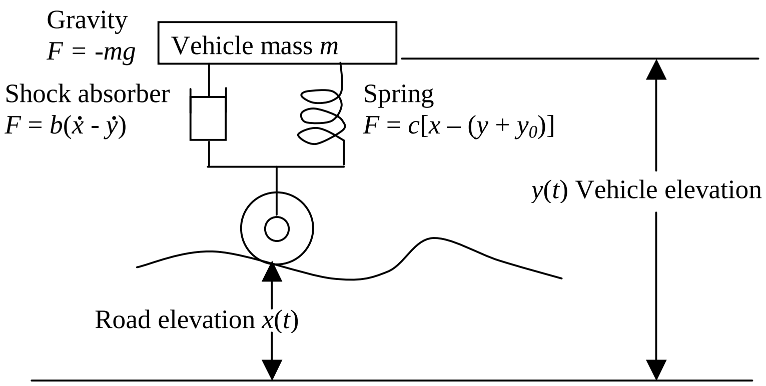
University of California at Berkeley
Department of Electrical Engineering and Computer Sciences
EECS 120, Professor J.M. Kahn
Midterm 2
Wednesday, November 13, 2002, 2:15-3:15 pm

Name: ____________________________________

Note: Indicate you answer clearly by circling it or drawing a box around it. When asked to make sketched of functions, label the horizontal and vertical axes of the plots.

Problem	Points	Score
1	30	
2	45	
3	25	
TOTAL:	100	

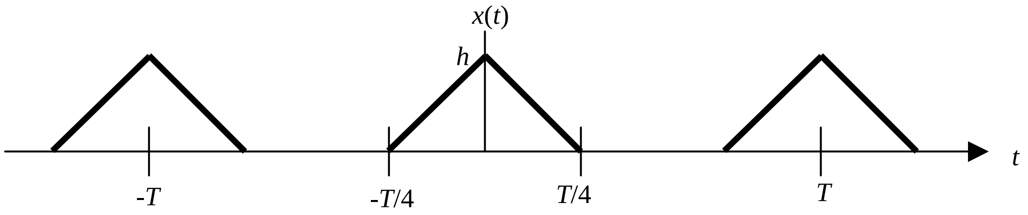
Problem 1 (30 pts.) The suspension of a car can be modeled as a second-order system with input $x(t)$ and output $y(t)$, as shown in the figure below. Here, y_0 is a constant, $m > 0$, $k > 0$, and $b \geq 0$.



It can be shown that the differential equation relating $x(t)$ and $y(t)$ is $m\ddot{y} + b\dot{y} + cy = b\dot{x} + cx$, where the dots denote time derivatives. When initial conditions are zero, the suspension can be considered an LTI system with input $x(t)$ and output $y(t)$.

(a) (10 pts.) Find the frequency response $H(j\omega)$ of this LTI system.

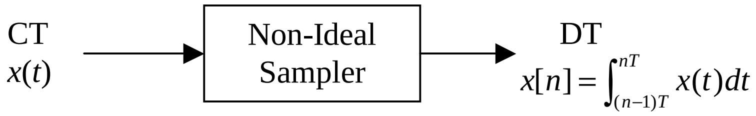
The car rolls along an infinitely long road with regularly spaced triangular speed bumps, so that $x(t)$ is the periodic signal indicated below.



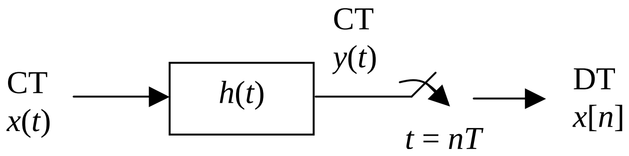
(b) (10 pts.) Find a Fourier series representation for $x(t)$.

(c) (10 pts.) Find an expression for the vehicle elevation $y(t)$ when the vehicle rolls over the road pictured above. This is most easily done using results obtained in parts (a) and (b). If you were unable to explicitly obtain $H(j\omega)$ and/or $X[k]$, you may give your answer to part (c) in terms of arbitrary $H(j\omega)$ and/or $X[k]$, in which case you may receive partial credit.

Problem 2 (45 pts.) A non-ideal sampler starts with a CT signal $x(t)$ and obtains a DT signal $x[n]$ according to the figure below.



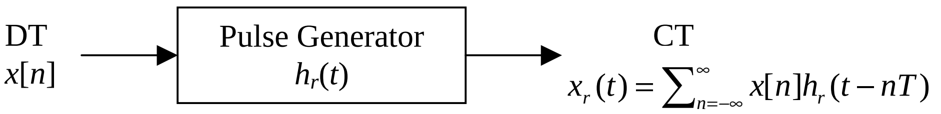
(a) (10 pts.) Show that the non-ideal sampling of $x(t)$ is equivalent to ideal sampling of $y(t) = x(t)*h(t)$, i.e., $x[n] = y(t)|_{t = nT}$, as indicated below, if we choose $h(t) = u(t) - u(t - T)$.



(b) (10 pts.) Let $x(t) \overset{FT}{\leftrightarrow} X(j\omega)$ and let $x[n] \overset{DTFT}{\leftrightarrow} X(e^{j\omega T})$. Find an expression for $X(e^{j\omega T})$ in terms of $X(j\omega)$ and T.

(c) (10 pts.) Suppose that $x(t) \overset{FT}{\leftrightarrow} X(j\omega)$ is bandlimited to $|\omega| < \omega_m$. Find the largest T such that $x[n] \overset{DTFT}{\leftrightarrow} X(e^{j\omega T})$ exhibits no aliasing.

(d) (15 pts.) Assume that the requirement in part (c) for no aliasing is satisfied. We reconstruct a CT signal $x_r(t)$ using the system shown. Find $h_r(t)$ such that $x_r(t) = x(t)$.



Problem 3 (25 pts.) A *Hilbert transformer* is an LTI system H_{HT} having impulse response $h_{HT}(t)$ and frequency response $H_{HT}(j\omega)$, which is given by:

$$H_{HT}(j\omega) = -j \cdot \text{sgn}(\omega) = \begin{cases} j & \omega < 0 \\ 0 & \omega = 0 \\ -j & \omega > 0 \end{cases}$$

If $x(t)$ is input ot the system, the output of $y(t) = H_{HT}\{x(t)\}$ is called the *Hilbert transform* of $x(t)$.

(a) (10 pts.) Let the input be $x(t) = \cos\omega_c t$. What is the output $y(t)$?

(b) (15 pts.) Find the impulse response $h_{HT}(t)$. *Hint:* use the fact that $\mathrm{sgn}(\omega) = 2u(\omega) - 1$.