EECS 120 **Fall 2004**

Midterm 1

• The exam is for one hour and 50 minutes.

• The maximum score is 100 points. The maximum score for each part of each problem is indicated.

• The exam is closed-book and closed-notes. Calculators, computing, and communication devices are NOT permitted.

• Two double-sided sheets of notes are allowed. These should be legible to normal eyesight, i.e. the lettering should not be excessively small.

• No form of collaboration between students is allowed.

1. *(10 points)* State whether the following are true or false. In each case, give a brief explanation. A correct answer without a correct explanation gets 2 points. A correct answer with a correct explanation gets 5 points.

   (a) Let $x(t)$ be a continuous time signal. Let $y_1(t)$, $y_2(t)$, and $y_3(t)$ denote the respective ouputs of a causal linear time invariant system to the inputs $x(t)$, $x^2(t)$, and $x^3(t)$. Then $y_3(t)$ can be determined from $y_1(t)$ and $y_2(t)$.

   (b) If $x[n]$ is a nonnegative sequence with discrete time Fourier transform $X(e^{j\omega})$, then

$$\sum_{n=-\infty}^{\infty} x[n] = \frac{1}{2\pi} \int_{2\pi} \mid X(e^{j\omega}) \mid d\omega \ .$$

2. *(10 points)*

   Let $x[n]$ and $y[n]$ be periodic with period 3, with

$$x[n] = \begin{cases} 1 & \text{if } n = -1 \\ 2 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \end{cases} \quad ,$$

   and

$$y[n] = \begin{cases} -1 & \text{if } n = -1 \\ 2 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \end{cases} \quad .$$

   Let $z[n]$ be the periodic sequence of period 3 that is the periodic convolution of $x[n]$ and $y[n]$, i.e.

$$z[n] = \sum_{l \in <3>} x[l] y[n-l] \ .$$

   Determine $z[n]$.

3. *(10 points)*

   Let

$$\Lambda(t) = \begin{cases} 1- \mid t \mid & \text{if } \mid t \mid \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   Let

$$x(t) = \sum_{n=-\infty}^{\infty} \Lambda(t-2n)\cos(\omega_0 t) \ .$$

   Find the Fourier transform of $x(t)$.

   Hint : The function

$$z(t) = \begin{cases} 1 & \text{if } \mid t \mid \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

   has Fourier transform

$$Z(j\omega) = \text{sinc}(\frac{\omega}{2\pi}) \ .$$

4. *(10 points)* Let $x[n]$ be a periodic sequence with period N. Assume $N = 3K$ for some integer K. Let a_k denote the discrete time Fourier series coefficients of $x[n]$. If $a_k = 0$ when k is not a multiple of 3, show that $x[n]$ must also be periodic with period K.

5. *(5 + 5 points)* Consider the causal linear time-invariant system whose input and output are related by the difference equation

$$y[n] + \frac{1}{3}y[n-1] = x[n] + x[n-2] - 3x[n-5] \ .$$

   (a) Find the transfer function of the system.

   (b) Find the output of this system for the input

$$x[n] = (-1)^n \text{ for all } n \ .$$

6. *(5 + 5 + 5 points)* Consider the continuous time system whose output $y(t)$ for the input $x(t)$ is given by

$$y(t) = x(t - (\int_t^{t+1} x(u)du)^2) \ .$$

   Is it :

   (a) linear ?

   (b) causal ?

   (c) BIBO stable ?

   In each case explain your answers briefly. There should be no ambiguity about which part of the problem you are answering. A correct answer with an incorrect explanation will get at most 2 points.

7. *(7 + 8 points)* Let

$$x_1(t) = \begin{cases} 1 & \text{if } \mid t \mid \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases},$$

and

$$x_2(t) = \begin{cases} t & \text{if } 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}.$$

(a) Sketch $y(t) = x_1(t) * x_2(t)$. You DO NOT need to write any formulas. The shape of your sketch of $y(t)$ should be accurate and the coordinates should be properly marked.

(b) Let

$$z_1(t) = \begin{cases} 1 & \text{if } 0 \leq t \leq 1 \\ 3 & \text{if } 1 < t \leq 2 \\ 0 & \text{otherwise} \end{cases},$$

and let

$$z_2(t) = \begin{cases} t & \text{if } 0 \leq t \leq 1 \\ \frac{1}{3}t - \frac{1}{3} \text{ if } 1 < t \leq 2 & \\ 0 & \text{otherwise} \end{cases}.$$

Let $w(t) = z_1(t) * z_2(t)$. Determine $w(t)$ in terms of $y(t)$ using basic properties of convolution. You need not determine $w(t)$ explicitly : just write it in terms of $y(t)$.

8. *(10 + 10 points)* Consider the function

$$x(t) = \begin{cases} 1 & \text{if } -1 < t \leq 0 \\ 1+t & \text{if } 0 < t \leq 1 \\ 2-t & \text{if } 1 < t \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the Fourier transform $X(j\omega)$ of the function $x(t)$.

(b) Let $y(t)$ be the periodic function of period 8 defined by

$$y(t) = \begin{cases} x(t - \frac{3}{2}) & \text{if } 0 < t \leq 4 \\ -x(t + \frac{5}{2}) & \text{if } -4 < t \leq 0 \end{cases}.$$

Find the Fourier series coefficients of $y(t)$.