EECS 120
Final Exam
Thur. Dec. 18, 2014
0810 - 1100 am

Name:____________________________
SID:_____________

- Closed book. Three single sided 8.5x11 inch pages of formula sheet. No calculators.
- There are 8 problems worth 200 points total. There may be more time efficient methods to solve problems.

Problem	Points	Score
1	26	
2	40	
3	12	
4	20	
5	36	
6	36	
7	30	
TOTAL	200	

In the real world, unethical actions by engineers can cost money, careers, and lives. The penalty for unethical actions on this exam will be a grade of zero and a letter will be written for your file and to the Office of Student Conduct.

$\tan^{-1} 0.1 = 5.7°$	$\tan^{-1} 0.2 = 11.3°$
$\tan^{-1} \frac{1}{2} = 26.6°$	$\tan^{-1} 1 = 45°$
$\tan^{-1} \frac{1}{3} = 18.4°$	$\tan^{-1} \frac{1}{4} = 14°$
$\tan^{-1} \sqrt{3} = 60°$	$\tan^{-1} \frac{1}{\sqrt{3}} = 30°$
$\sin 30° = \frac{1}{2}$	$\cos 60° = \frac{\sqrt{3}}{2}$
$\cos 45° = \frac{\sqrt{2}}{2}$	$\sin 45° = \frac{\sqrt{2}}{2}$

$20 \log_{10} 1 = 0dB$	$20 \log_{10} 2 = 6dB$	$\pi \approx 3.14$
$20 \log_{10} \sqrt{2} = 3dB$	$20 \log_{10} \frac{1}{2} = -6dB$	$2\pi \approx 6.28$
$20 \log_{10} 5 = 20db - 6dB = 14dB$	$20 \log_{10} \sqrt{10} = 10$ dB	$\pi/2 \approx 1.57$
$1/e \approx 0.37$	$\sqrt{10} \approx 3.164$	$\pi/4 \approx 0.79$
$1/e^2 \approx 0.14$	$\sqrt{2} \approx 1.41$	$\sqrt{3} \approx 1.73$
$1/e^3 \approx 0.05$	$1/\sqrt{2} \approx 0.71$	$1/\sqrt{3} \approx 0.58$

$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ $\qquad$ $\sin 2\theta = 2 \sin \theta \cos \theta$

$e^{j\theta} = \cos \theta + j \sin \theta$ $\qquad$ $\sin^2 \theta + \cos^2 \theta = 1$

Problem 1 LTI Properties (26 pts)

[24 pts] Classify the following systems, with input $x(t)$ (or $x[n]$) and output $y(t)$ (or $y[n]$). In each column, write "yes", "no", or "?" if the property is not decidable with the given information. (+1 for correct, 0 for blank, -0.5 for incorrect). (For 1d, you are given the system is known to be linear and time=invariant.) For 1b and 1d, 2 test input cases are given.

Let $\Pi(t) = u(t+\frac{1}{2}) - u(t-\frac{1}{2})$

System	Causal	Linear	Time-invariant	BIBO stable
a. $y(t) = 2x(t-1) - 5$				
b. If $x(t) = 0 \rightarrow y(t) = 0$. If $x(t) = u(t-1) \rightarrow y(t) = tu(t)$.				
c. $y(t) = x(t)[\cos(2\pi t)u(t)]$				
d. If $x(t) = 0 \rightarrow y(t) = 0$ If $x(t) = u(t) \rightarrow y(t) = u(t-1)$		YES	YES	
e. $y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\Pi(t-\tau)d\tau$				
f. $y(t) = x(t) \cdot [1 - \delta(t+100)]$				
g. $y[n] = \mathcal{Z}^{-1}\{\frac{z^2}{z+1}\} * x[n]$				

Problem 2 Short Answers (40 pts)
Answer each part independently. Note $\Pi(t) = u(t+\frac{1}{2}) - u(t-\frac{1}{2})$.

[4 pts] a. Complete the table with the appropriate type of Fourier transform to use (FS, FT, DTFT, or DFT) on a signal of each type.

	aperiodic in time	periodic in time
continuous time		
discrete time		

[3 pts] b. $X(j\omega) = \cos(\omega/2) + 1$. Find $x(t)$.

$x(t) =$ __________________

[4 pts] c. A periodic signal $x(t) = p(t) * \sum_{n=-\infty}^{\infty} \delta(t-2n)$, where $\mathcal{F}\{p(t)\} = P(j\omega) = \cos(\omega/2) + 1$. The fundamental period $\omega_o = \pi$. Find the Fourier series coefficients a_k.

$a_k =$ __________________

[7 pts] d. A periodic signal $x(t)$ has period 4 seconds and Fourier Series coefficients $a_k = \frac{\sin(k\pi/4)}{k\pi}$. Find the time average power $\frac{1}{T}\int_T x^2(t)dt$.

time average power = __________________

[9 pts] e. Initial and final value.

i. Given $X(s) = \frac{s+3}{s^2+3s+2}$. Find $x(0^+) =$________________

ii. Given causal $X(z) = \frac{z^{-2}+2z^{-3}}{1-2z^{-1}+\frac{5}{4}z^{-2}-\frac{1}{4}z^{-3}}$.

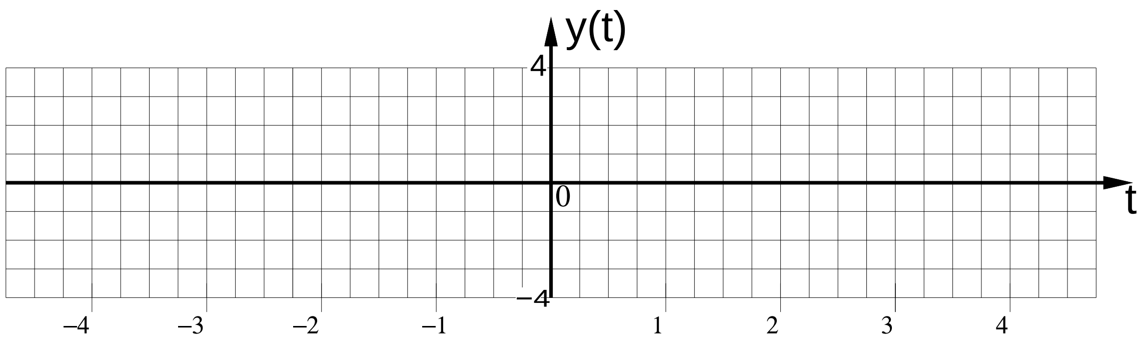
Find $\lim_{n\to\infty} x[n] =$ ________________

iii. Given causal $X(z) = \frac{2+3z^{-1}}{1-2z^{-1}+\frac{5}{4}z^{-2}-\frac{1}{4}z^{-3}}$.

Find $x[0] =$ ________________

[5 pts] f. Given causal $X(s) = \frac{s+5}{s^3+3s^2+2s}$. Find $x(t) =$ ________________

[8 pts] g. Sketch $y(t) = 3\pi \cdot u(t+1) * \cos(\pi t)u(t)$

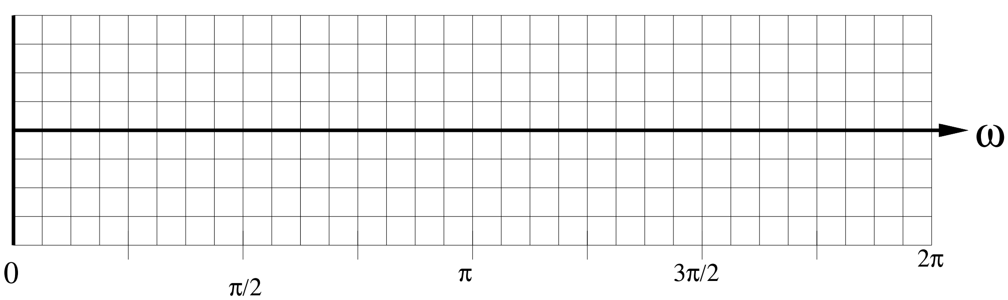


Problem 3. Digital Filter (12 pts)
A continuous time filter has impulse response $h(t) = e^{-\pi t/2}u(t)$.

[6 pts] a. The filter is sampled such that $h[n] = h(nT_s)$ where the sampling rate $T_s = 1$ sec. Find the Z transform of $h[n]$.

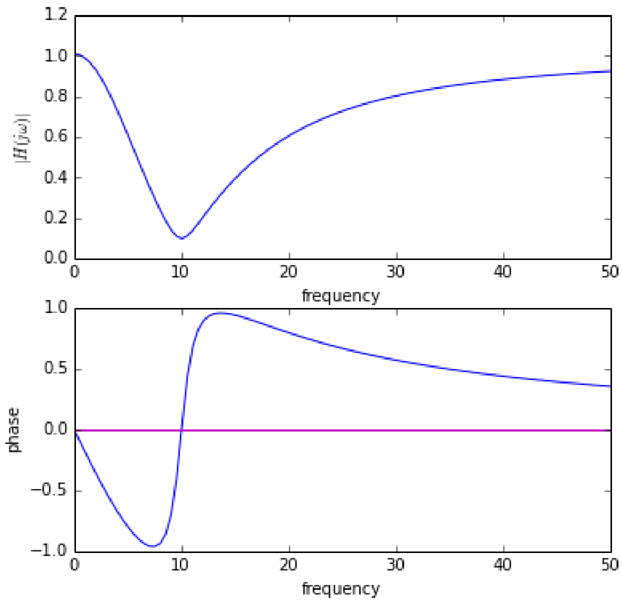
$H(z) =$ __________

[6 pts] b. Sketch $|H(e^{j\omega})|$, labelling maximum and minimum amplitude. (Maximum and minimum may be left as functions of e).

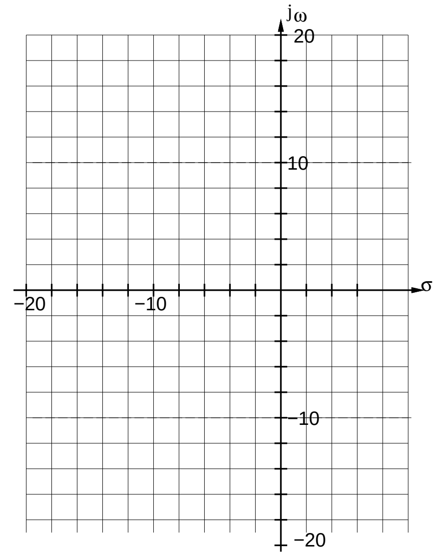


Problem 4. CT and Digital Filters (20 pts)

[10 pts] a. The magnitude and phase response for a continuous time, real, causal, stable LTI system is shown below. Sketch a pole-zero diagram for a stable system (using minimum number of poles and zeros) which would match the given magnitude and phase response.

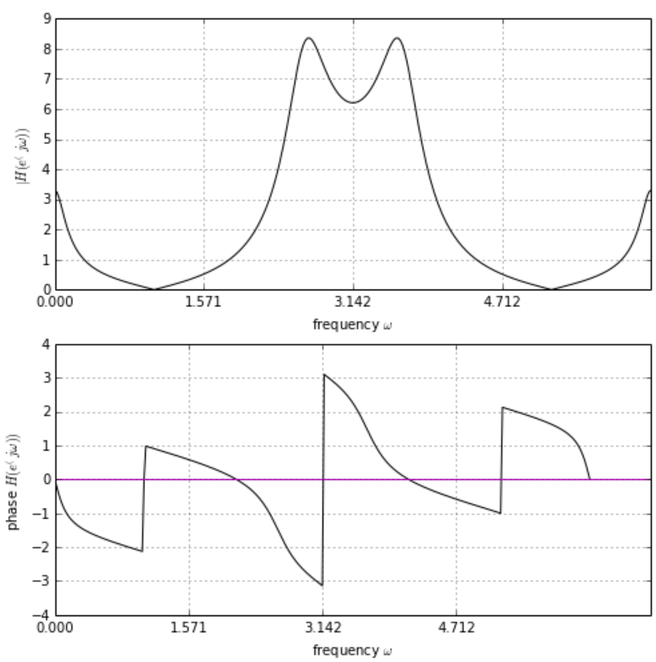


Given magnitude and phase.

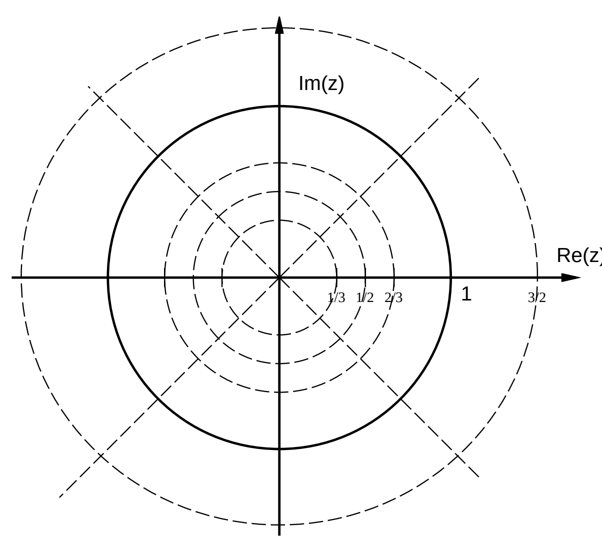


Sketch corresponding pole-zero plot here.

[10 pts] b. The magnitude and phase response for a discrete time, real, causal, stable LTI system is shown below. Sketch a pole-zero diagram for a stable system (using minimum number of poles and zeros) which would match the given magnitude and phase response. (Note: the phase change at π is just numerical wrapping.)



Given magnitude and phase.



Sketch corresponding pole-zero plot here.

Problem 5. Z transform (36 pts)

Consider a causal DT system with

$$H(z) = \frac{z^2 + 9/4}{z(z - \frac{1}{2})}$$

.

[4 pts]. a. Find the unit sample response $h[n]$ for $H(z)$.

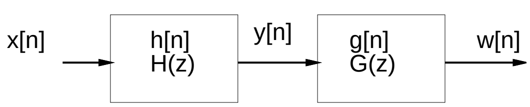
$h[n] =$______________

[4 pts] b. With input $x[n]$ and output $y[n]$, find the linear difference equation in terms of $y[n]$ for this system:

$y[n] =$______________

[4 pts] c. $H(z)$ is not minimum phase. Find a minimum phase function $F(z)$ such that $|H(e^{j\omega})| = |F(e^{j\omega})|$ for all ω.

$F(z) =$______________

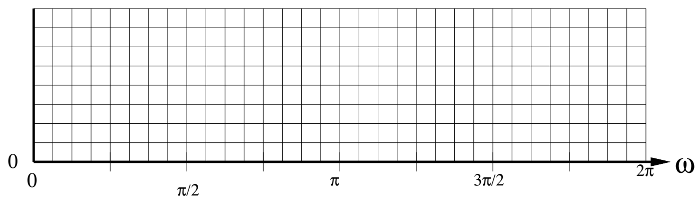


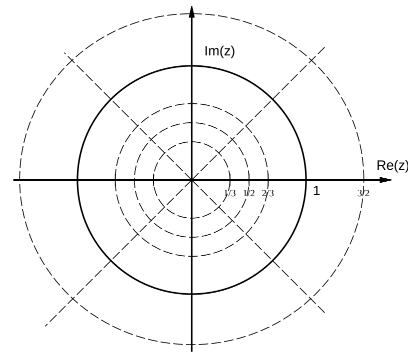
[4 pts] d. Find a **stable** $G(z)$ such that $|H(e^{j\omega})G(e^{j\omega})| = 1$ for all ω.

$G(z) =$______________

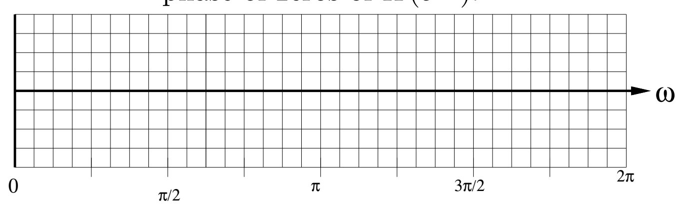
[12 pts] e. VERSION 2 Approximately sketch $|H(e^{j\omega})|$ [4 pts] and phase of the zeros *only* of $H(e^{j\omega})$ [8 pts] on the plots below, noting key maxima and minima.

$|H(e^{j\omega})|$:





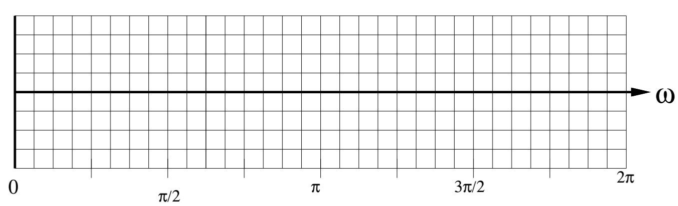
phase of zeros of $H(e^{j\omega})$:

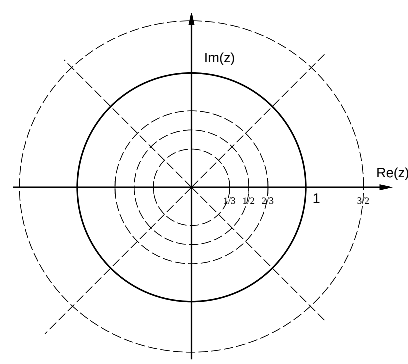


scratch work area

[8 pts] f. VERSION 2 Sketch the phase due to the zeros of $F(e^{j\omega})$ on the plot below, noting key maxima and minima. (Hint: sketch phase from each zero independently, then add.)

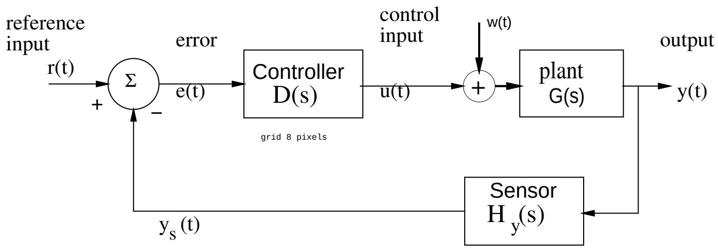
$\angle F(e^{j\omega})$:





scratch work area

Problem 6. Control (36 pts)



[2 pts] a. Find the transfer function $\frac{E(s)}{W(s)}$ in terms of D, G, H_y.

$\frac{E(s)}{W(s)} =$ ____________________

[4 pts] b. Find the transfer function $\frac{Y(s)}{W(s)}$ in terms of D, G, H_y.

$\frac{Y(s)}{W(s)} =$ ____________________

[10 pts] c. For the system above, let $D(s) = k_p$, $H_y(s) = 1$, and $G(s) = \frac{2\pi}{s+2\pi}$.

With input $r(t) = r_o u(t)$, and step disturbance $w(t) = w_o u(t)$ determine trend of $y(t)$ as $t \to \infty$.

$y(t) \to$ ____________________

[10 pts] d. For the system above, let $D(s) = k_p$, $H_y(s) = 1$, and $G(s) = \frac{2\pi}{s+2\pi}$.

With input $r(t) = 0$, and disturbance $w(t) = \cos(2\pi t)u(t)$, determine the sinusoidal steady state response for $y(t)$ after transients have decayed. (Hint: $y(t)$ will be of the form $M\cos(2\pi t + \phi)$. Determine M and ϕ.)

$y(t) \approx$ ________________

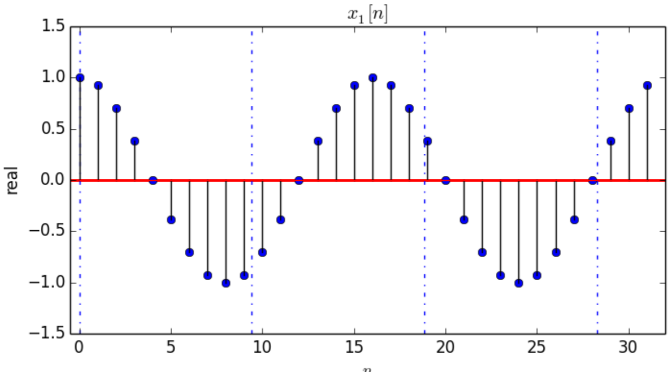
[10 pts] e. For the system above, let $D(s) = \frac{k_p + k_d s}{s^2 + 4\pi^2}$, $H_y(s) = 1$, and $G(s) = \frac{2\pi}{s+2\pi}$.

With input $r(t) = 0$, and disturbance $w(t) = \cos(2\pi t)u(t) + 0.5u(t)$, determine the steady state response for $y(t)$ after transients have decayed.

$y(t) \approx$ ________________

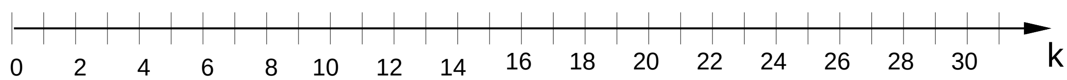
Problem 7. DFT problem (30 pts)

[10 pts] a. Given $x_1[n] = \cos(2\pi \frac{n}{16})$ as shown:

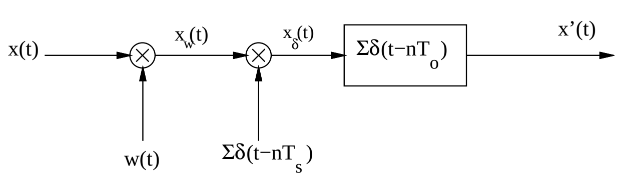


sketch $X_1[k]$, the 32 point DFT of $x_1[n]$, labelling amplitudes.

$X_1[k]$:



The equivalent signal processing operations for a windowed DFT can be represented by the following block diagram:

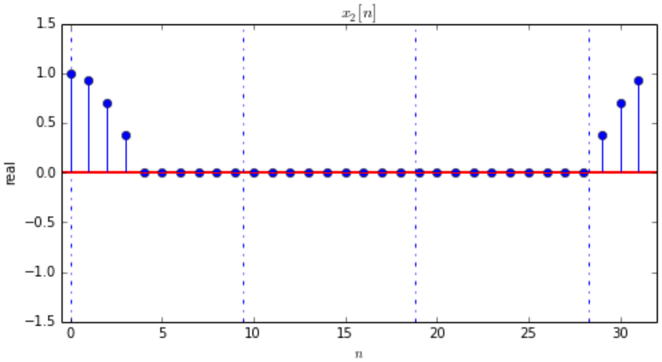


[2 pts] b. For $x_1[n]$ as given above, what are possible T_s and T_o?

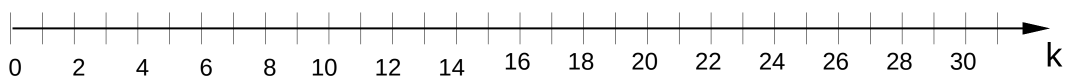
$T_s =$ _____ $T_o =$ _____

[2 pts] c. For $X_1[k]$, and your T_o above, what is the equivalent spacing of the frequency samples?
spacing = _____ radians/second

[16 pts] d. Given $x_2[n] = \cos(2\pi \frac{n}{16}) w[n]$ as shown:



sketch $X_2[k]$, the 32 point DFT of $x_2[n]$, approximately labelling key amplitudes.



$X_2[k]$:

Area for scratch work.