EECS 120 Final Exam

Thu. Dec. 15, 2016: 810 - 1100 am

Name:____________________________ person left:____________________________

SID:______________________ person right:____________________________

- Closed book. Two double sided 8.5x11 inch pages formula sheet. No calculators.
- There are 7 problems worth 200 points total. There may be more time efficient methods to solve problems.

  In the real world, unethical actions by engineers can cost money, careers, and lives. The penalty for unethical actions on this exam will be a grade of zero and a letter will be written for your file and to the Office of Student Conduct.

Problem	Points	Score
1	32	
2	30	
3	20	
4	30	
5	28	
6	20	
7	40	
TOTAL	200	

Tables for reference:

$\tan^{-1}\frac{1}{10} = 5.7°$	$\tan^{-1}\frac{1}{5} = 11.3°$
$\tan^{-1}\frac{1}{4} = 14°$	$\tan^{-1}\frac{1}{3} = 18.4°$
$\tan^{-1}\frac{1}{2} = 26.6°$	$\tan^{-1}\frac{1}{\sqrt{3}} = 30°$
$\tan^{-1} 1 = 45°$	$\tan^{-1}\sqrt{3} = 60°$
$\sin 30° = \frac{1}{2}$	$\cos 30° = \frac{\sqrt{3}}{2}$
$\cos 45° = \frac{\sqrt{2}}{2}$	$\sin 45° = \frac{\sqrt{2}}{2}$

$20\log_{10} 1 = 0dB$	$20\log_{10} 2 = 6dB$	$\pi \approx 3.14$
$20\log_{10}\sqrt{2} = 3dB$	$20\log_{10}\frac{1}{2} = -6dB$	$2\pi \approx 6.28$
$20\log_{10} 5 = 20db - 6dB = 14dB$	$20\log_{10}\sqrt{10} = 10$ dB	$\pi/2 \approx 1.57$
$1/e \approx 0.37$	$\sqrt{10} \approx 3.164$	$\pi/4 \approx 0.79$
$1/e^2 \approx 0.14$	$\sqrt{2} \approx 1.41$	$\sqrt{3} \approx 1.73$
$1/e^3 \approx 0.05$	$1/\sqrt{2} \approx 0.71$	$1/\sqrt{3} \approx 0.58$

Problem 1 LTI Properties (32 pts)

[32 pts] Classify the following systems, with input $x(t)$ (or $x[n]$) and output $y(t)$ (or $y[n]$). In each column, write "yes", "no", or "?" if the property is not decidable with the given information. (+1 for correct, 0 for blank, -0.5 for incorrect).

Let $\Pi(t) = u(t+\frac{1}{2}) - u(t-\frac{1}{2})$

System	Causal	Linear	Time -invariant	BIBO stable
a. $y(t) = x(t-1) + e^{-t}u(t-1)$				
b. $y(t) = x(t-1) * e^{-t}u(t-1)$				
c. $y(t) = x(t-1) \cdot e^{-t}u(t-1)$				
d. $y[n] = x[n] * \cos(2\pi n)u[n]$				
e. $y(t) = \sum_{n=0}^{\infty} x(t)(\frac{1}{2})^n \delta(t-n/10)$				
f. $y(t) =$ $\int_{\tau=0}^{\infty} x(t-\tau)(\sum_{n=0}^{\infty}(\frac{1}{2})^n \delta(\tau - n/10))d\tau$				
g. $y(t) = x(t) * (u(t-2) - u(t+2))$				
h. Given system $H\{\}$ such that $H\{x(t)\} = y(t)$ $H\{a_1x_1(t) + a_2x_2(t)\} = a_1H\{x_1(t)\} + a_1H\{x_1(t)\}$ $\forall a_1, a_2, x_1(t), x_2(t)$ and $H\{\delta(t)\} = \cos(2\pi t + \pi/4)u(t)$				

Problem 2 Short Answers (30 pts)

Answer each part independently. Note $\Pi(t) = u(t + \frac{1}{2}) - u(t - \frac{1}{2})$.

[2 pts] a. Given $x(t) = \delta(t-2) + \delta(t+2) + 2\delta(t)$. Find $X(j\omega)$.

$X(j\omega) =$__________________

[4 pts] b. A periodic signal $x(t) = p(t) * \sum_{n=-\infty}^{\infty} \delta(t - 10n)$, where $p(t) = \Pi(t)$.

Find the Fourier series coefficients a_k, such that $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t}$, where $\omega_o = \frac{2\pi}{10}$.

$a_k =$__________________

[6 pts] c. A signal $x(t) = \cos(2\pi t)$ is passed through an LTI system with impulse response $h(t) = e^{-t}u(t)$.

Find the output of the system, $y(t)$ (express as a real function, and not as an integral).

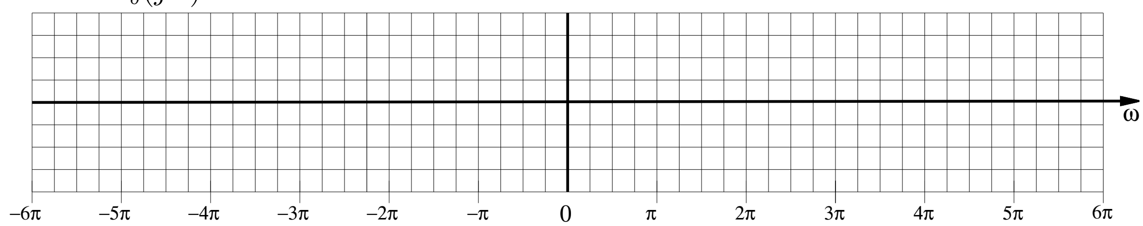
$y(t) =$__________________

Problem 2 Short Answers, cont.

[4 pts] d. A time signal $x(t)$ has FT $X(j\omega) = \Pi(\frac{\omega}{2\pi})$.

$x(t)$ is sampled such that $x_\delta(t) = x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - \frac{4n}{3})$.

Sketch $X_\delta(j\omega)$



[4 pts] e. Initial and final value.

Given $X(s) = \frac{s+3}{s^3+2s^2+2s}$.

Find $x(0^+) =$__________________. Find $\lim_{t\to\infty} x(t) =$ __________________

[5 pts] f. Given causal $X(s) = \frac{s^2+11s+30}{s^2+5s+6}$. Find $x(t) =$ __________________

[5 pts] g. A system with input $x(t)$ and output $y(t)$ is described by the following differential equation:

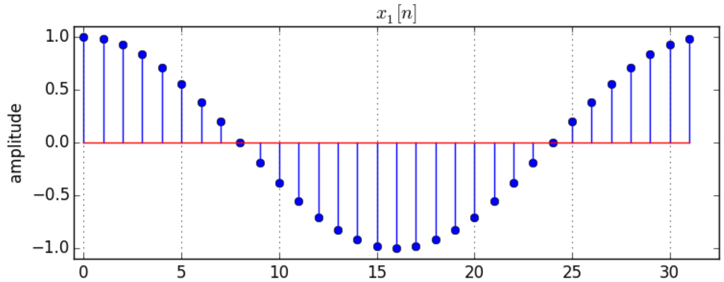
$\frac{d^2}{dt^2}y + 3\frac{d}{dt}y + 2y = \frac{d}{dt}x + 3x$.

Assuming zero input ($x(t) = 0$ with $y(0^-) = 4$ and $\frac{d}{dt}y(0^-) = 1$,
find the output $y(t)$ for $t \geq 0$.

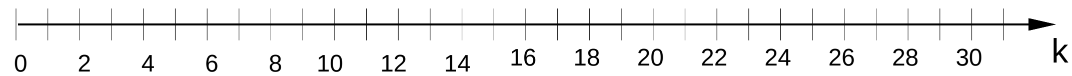
$y(t) =$ __________

Problem 3. Discrete Fourier Transform (20 pts)

[8 pts] a. Given $x_1[n] = \cos(2\pi \frac{n}{32})$ as shown:

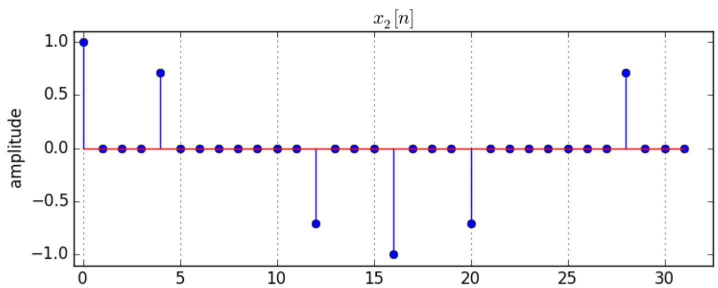


sketch $X_1[k]$, the 32 point DFT of $x_1[n]$, labelling amplitudes.

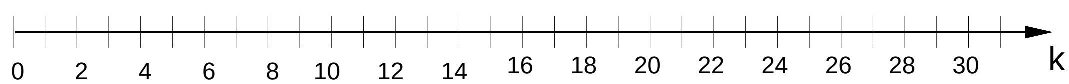


$X_1[k]$:

[12 pts] b. Given $x_2[n] = x_1[n] \cdot p[n]$, where $p[n] = \sum_{m=0}^{7} \delta[n - 4m]$, as shown:



sketch $X_2[k]$, the 32 point DFT of $x_2[n]$, labelling amplitudes.(Hint: down sample.)



$X_2[k]$:

Problem 4. DTFT and Up/Down Sample (30 pts)

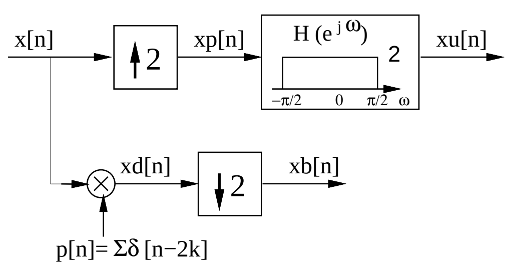
[4 pts] a. Let $v[n] = (\frac{4\sin \pi n/4}{\pi n})$. Find the DTFT $V(e^{j\Omega})$.

$V(e^{j\Omega}) =$__________________

[4 pts] b. Let $x[n] = (\frac{4\sin \pi n/4}{\pi n})^2$. Find the DTFT $X(e^{j\Omega})$.

$X(e^{j\Omega}) =$__________________

A sampled signal $x[n] = (\frac{4\sin \pi n/4}{\pi n})^2$ is upsampled to $x_u[n]$ and downsampled to $x_b[n]$ as shown in the block diagram below.

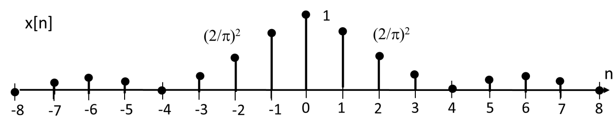


c,d,e,f,g: For each stage in the block diagram, sketch the sampled signal and the DTFT, noting heights and widths on the next page.

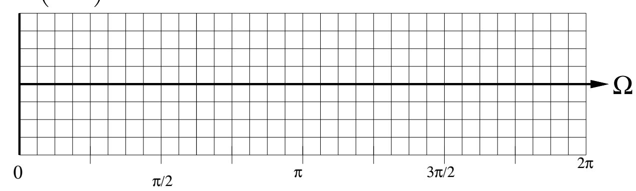
Problem 4. Up/Down Sample, cont.

[2 pts] c.

$x[n]$

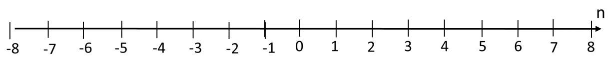


$X(e^{j\Omega})$

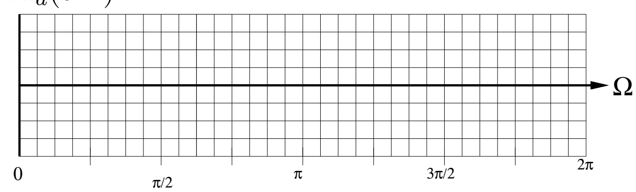


[5 pts] d.

$x_d[n]$

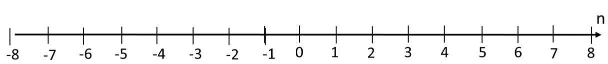


$X_d(e^{j\Omega})$

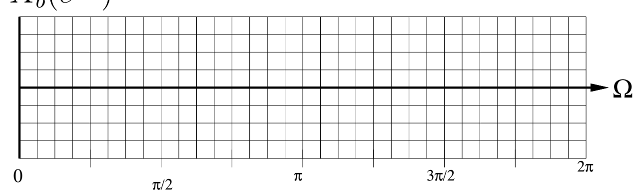


[5 pts] e.

$x_b[n]$

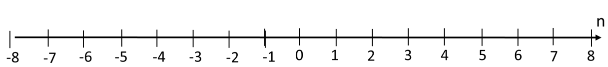


$X_b(e^{j\Omega})$

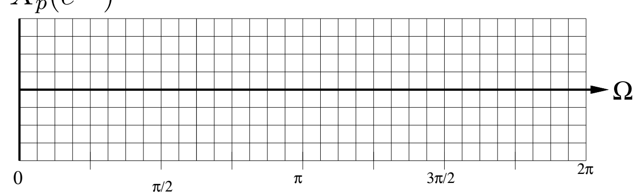


[5 pts] f.

$x_p[n]$

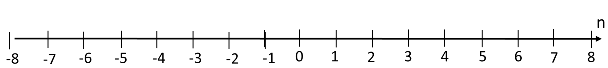


$X_p(e^{j\Omega})$

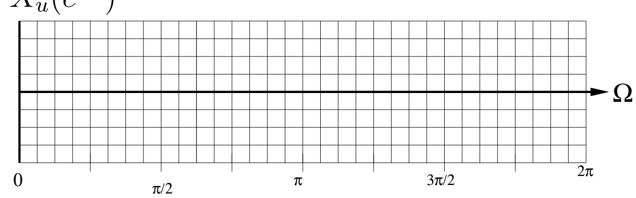


[5 pts] g.

$x_u[n]$



$X_u(e^{j\Omega})$



Problem 5. Digital Filter (30 pts)

Consider a causal DT system with

$$H(z) = \frac{z + \frac{3}{2}}{z^2 - \frac{1}{4}}$$

.

[4 pts]. a. Find the unit sample response $h[n]$ for $H(z)$.

$h[n] =$______________

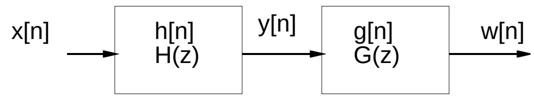
[4 pts] b. With input $x[n]$ and output $y[n]$, find the linear difference equation in terms of $y[n]$ for this system:

$y[n] =$______________

[4 pts] c. $H(z)$ is not minimum phase. Find a minimum phase function $F(z)$ such that $|H(e^{\jmath\Omega})| = |F(e^{\jmath\Omega})|$ for all Ω.

$F(z) =$______________

Problem 5. Digital Filter, cont.

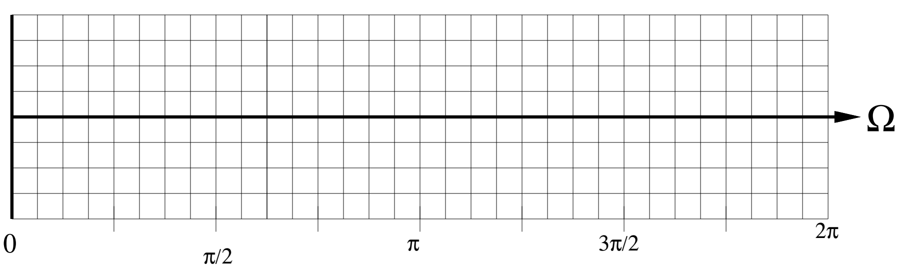


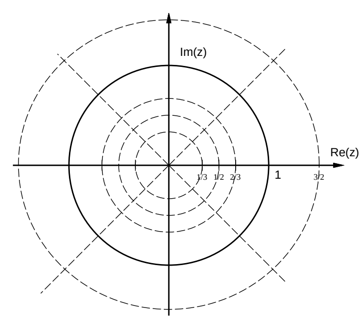
[4 pts] d. Find a **stable** $G(z)$ such that $|H(e^{j\Omega})G(e^{j\Omega})| = 1$ for all Ω.

$G(z) =$______________

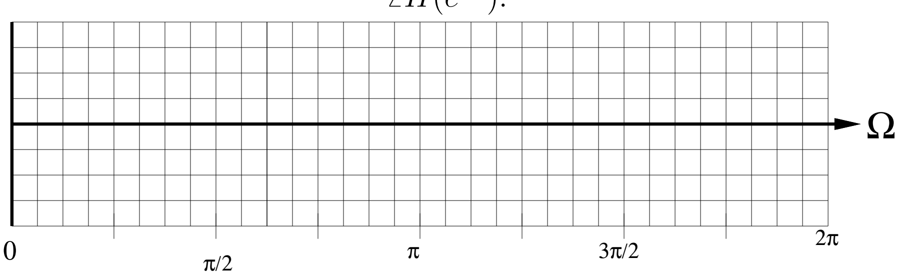
[12 pts] e. Approximately sketch $|H(e^{j\Omega})|$ [4 pts] and $\angle H(e^{j\Omega})$ [8 pts] on the plots below, noting key maxima and minima.

$|H(e^{j\Omega})|$:



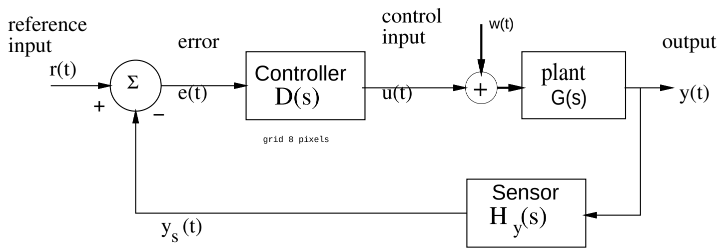


$\angle H(e^{j\Omega})$:



scratch work area

Problem 6. Control (20 pts)



Let $H_y(s) = 1$, $D(s) = k$, $w(t) = 0$, and

$$G(s) = \frac{1/\sqrt{3}}{(s+1)^2(s+\sqrt{3})(s+\frac{\sqrt{3}}{3})}$$

.

[5 pts] a. Find the closed loop transfer function $\frac{Y(s)}{R(s)}$

$\frac{Y(s)}{R(s)} =$ ____________________

[6 pts] b. Determine the frequency ω_{gm} such that $\angle G(\omega_{gm}) \approx -180°$.

$\omega_{gm} =$ ____________________

[6 pts] c. What is the gain margin for the closed-loop system? That is, what is the maximum value of k for which the system will be stable?

$k \leq$ ____________________

[3 pts] d. At the maximum value k found above, the system will be marginally stable. At what frequency would you expect the system to oscillate?

$\omega_{osc} =$ ____________________

Problem 7. Smart home temperature control (40 pts)

With the innovations in the internet of things and ubiquitous sensing, more and more homes are equipped with smart devices. Alex recently bought a smart heater system for his home – excited and curious, he decides to explore how it works.

He first draws out a diagram using what he learned in EE120 (**Fig. 1**). The smart heater has three basic components: the thermometer ($H_y(s)$), the controller ($D(s)$), and the heater component ($C(s)$). The temperature in the house $y(t)$ is continuously monitored by the thermometer, $y_s(t)$. The smart heater automatically decides the set point temperature $r(t)$ based on whether Alex is at home or not, and sends a continuous control signal $u(t)$ to the heater.

The heater is able to produce **heat at rate** $v(t)$ (Watt). Also Alex figured that heat can be also dissipated through the wall at rate $w(t)$ (Watt). Based on thermodynamics, he modeled the temperature $y(t)$ in the home by $\frac{dy(t)}{dt} = v(t) - w(t)$. Throughout the questions, assume $H_y(s) = 1$.

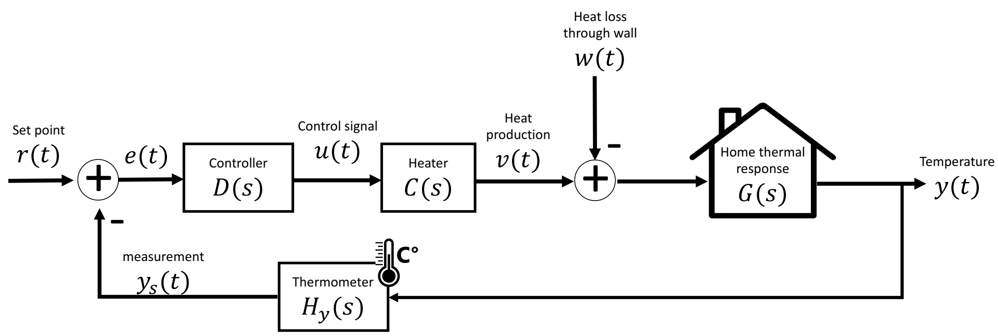


Fig. 1 Smart heater to regulate temperature

[4 pts] a. Determine $G(s)$. You can assume $y(0) = 0$ for the initial condition.

$G(s) =$

[9 pts] b. Determine the transfer functions from input $r(t)$ to $y(t)$ and input $w(t)$ to $y(t)$ (considering each independently). Express your answer in terms of $D(s), C(s)$. Do they have the same poles?

$\frac{Y(s)}{R(s)} =$ _______________ $\frac{Y(s)}{W(s)} =$ _______________

Problem 7. Smart home temperature control, cont.

[6 pts] c. Suppose the transfer function for the heater is $C(s) = \frac{k_c}{1+sQ}$. To find k_c and Q, Alex sends a unit step change in the control signal $u(t)$ and measured the step response in $v(t)$, shown below. Determine the constants k_c and Q. You can use $e^{-1} \approx 0.37$.

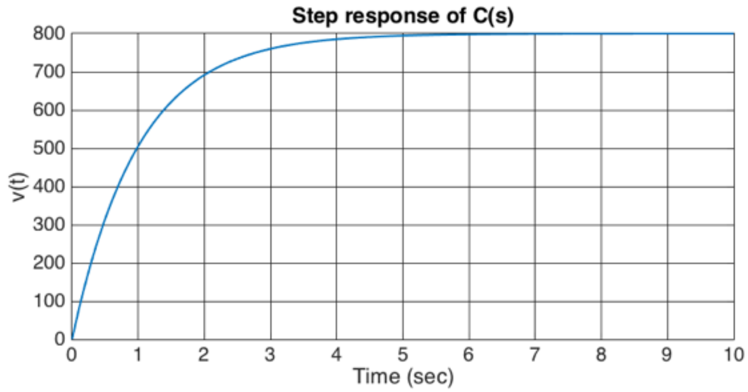
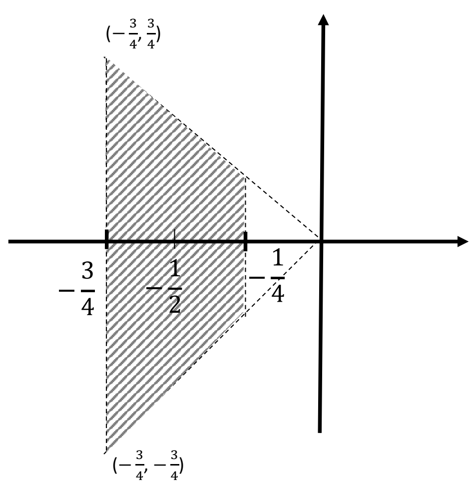


Fig. 2 Step response of $C(s)$

$k_c =$ ______________ $Q =$ ______________

[4 pts] d: Suppose $D(s)$ is proportional control, that is, $D(s) = K$. What is the largest K the controller can have if all poles of the closed loop system lie in the area shown in Fig. below?



$K <$ ______________

Problem 7. Smart home temperature control, cont.

[4 pts] e: Suppose the heat dissipation $w(t)$ is a unit step function. How large will the error $e(t)$ be in steady state with control in the previous part due to the heat dissipation?

$\lim_{t\to\infty} e(t) =$ ______________

[8 pts] f. Suppose the controller $D(s)$ is a PI controller: $u(t) = k_p e(t) + \frac{k_p}{T_i}\int_0^t e(t)dt$ (note here $u(t)$ is the control signal in Fig. 1). Alex found a way to keep $r(t) = y_s(t) + 1$, so the control error $e(t)$ increased as a step function: $e(t) = 1$ for $t > 0$ and 0 otherwise. By comparing the observed control signal output $u(t)$ (Fig. 3) with the theoretical output, please find out the parameters k_p, T_i of the PI controller $D(s)$.

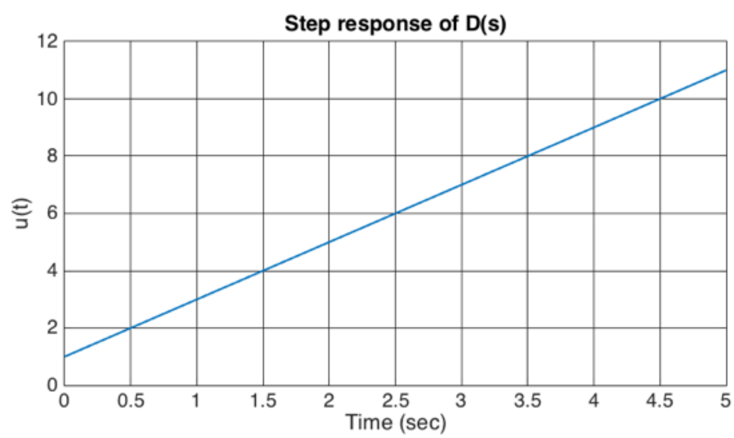


Fig. 3 Step response of $D(s)$

$k_p =$ ______________ $T_i =$ ______________

[5 pts] g. Suppose the heat dissipation $w(t)$ is a unit step function. How large will the error $e(t)$ be in steady state with the identified PI controller due to the heat dissipation?

$\lim_{t\to\infty} e(t) =$ ______________