University of California at Berkeley
Department of Electrical Engineering and Computer Sciences
Professor J. M. Kahn, EECS 120, Fall 1998
Final Examination, Wednesday, December 16, 1998, 5-8 pm

NAME: ____

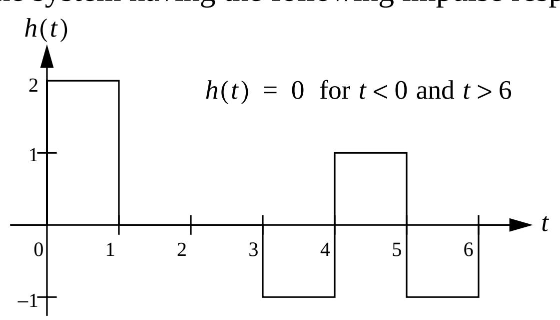
Note to students in Fall 99:

Only problems 1, 2, 5, 6 would be appropriate for your Midterm 2.

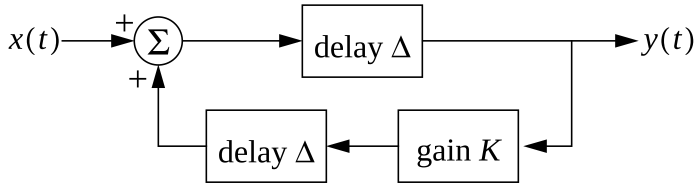
Problem 1 (15 pts.) The *equivalent noise bandwidth* of a CT LTI system $h(t) \overset{FT}{\leftrightarrow} H(j\omega)$ is defined as:

$$\text{BW} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{|H(j\omega)|^2}{|H(j0)|^2} d\omega.$$

Find the value of BW for the system having the following impulse response.



Problem 2 (25 pts.) The CT LTI system shown here can model many simple situations that produce echoes. Here, K and Δ are real constants, and $\Delta \geq 0$.



(a) (10 pts.) Find an expression for the impulse response $h(t)$.

(b) (10 pts.) For what values of K is the system stable? Justify your answer.

(c) (5 pts.) Let $K = -1$, and assume $x(t) = u(t) - u(t - 2\Delta)$. Sketch $y(t)$.

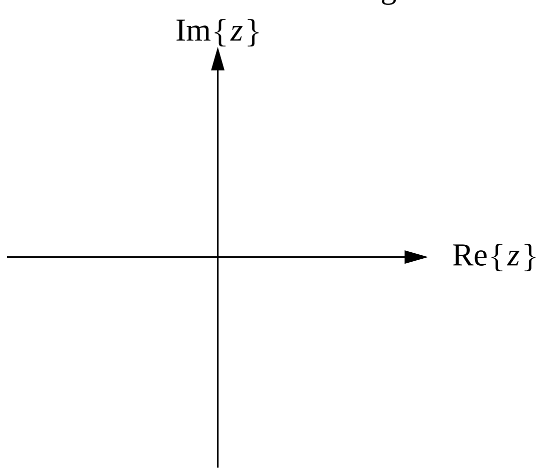
Problem 3 (40 pts.) Consider a DT system described by the difference equation:

$$y[n] + y[n-1] - 2y[n-2] = x[n] + x[n-1] .$$

(a) (5 pts.) Sketch a realization of this system using only two delay elements.

(b) (5 pts.) Find the transfer function $H(z)$.

(c) (5 pts.) Plot the poles and zeros and indicate the region of convergence of $H(z)$.



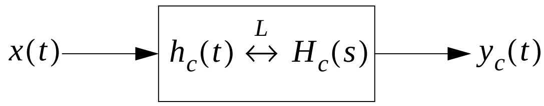
(d) (5 pts.) Is the system BIBO stable? Justify your answer.

(e) (5 pts.) Find the impulse response $h[n]$.

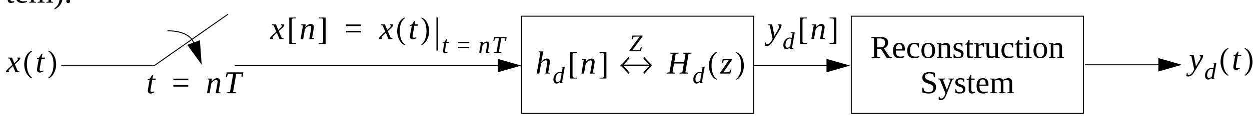
(f) (5 pts.) Let the input be $x[n] = 3^n$, $\forall n$. Find $y[n]$, $\forall n$.

(g) (10 pts.) Let the input be $x[n] = 3^n u[n]$, $\forall n$. (If you are interested in the initial conditions, they are fully specified by our specification of the input signal.) Find $y[n]$, $\forall n$. Explain why, as $n \to \infty$, $y[n]$ doesn't agree with the result found in part (f).

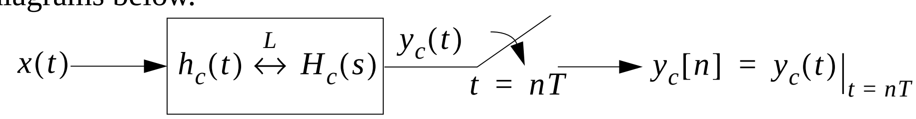
Problem 4 (45 pts.) In the diagram below, we process a CT signal $x(t)$ using a CT LTI system $h_c(t) \overset{L}{\leftrightarrow} H_c(s)$ to obtain the output $y_c(t)$. (Here, the subscript *c* means that $y_c(t)$ was processed using a CT system).

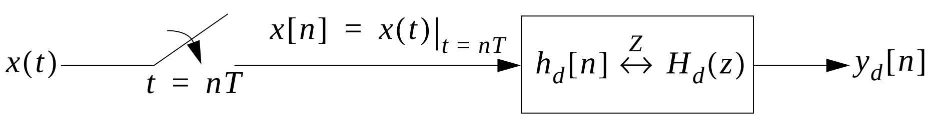


Alternatively, we can sample $x(t)$ at some appropriate rate $1/T$ to obtain a DT signal $x[n]$, process this using a DT LTI system $h_d[n] \overset{Z}{\leftrightarrow} H_d(z)$ to obtain $y_d[n]$, and perform reconstruction to obtain the CT signal $y_d(t)$. (Here, the subscript *d* means that $y_d(t)$ was processed using a DT system).



How to choose $h_d[n] \overset{Z}{\leftrightarrow} H_d(z)$ so that its effect is similar to that of $h_c(t) \overset{L}{\leftrightarrow} H_c(s)$ is a major topic in the study of digital signal processing. Here, we consider one technique, which makes reference to the diagrams below.





We choose some “typical” input signal $x(t)$, and then choose $h_d[n] \overset{Z}{\leftrightarrow} H_d(z)$ so that $y_d[n] = y_c[n]$. It should be emphasized that given the resulting $h_d[n] \overset{Z}{\leftrightarrow} H_d(z)$, the condition $y_d[n] = y_c[n]$ is satisfied only for that particular choice of $x(t)$. **In this problem, we choose $x(t) = u(t)$, which is a common choice, and we say that $h_d[n] \overset{Z}{\leftrightarrow} H_d(z)$ has been obtained from $h_c(t) \overset{L}{\leftrightarrow} H_c(s)$ by a “step-invariant transformation”.** (We are not concerned by the fact that the unit step function is not strictly bandlimited.)

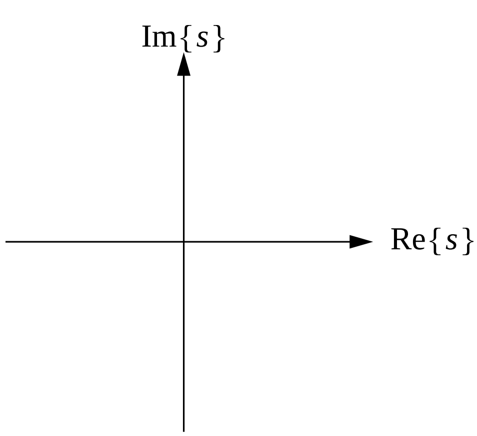
(a) (5 pts.) Consider a causal CT system with impulse response $h_c(t)$. Show that the step-invariant $h_d[n] \overset{Z}{\leftrightarrow} H_d(z)$ are specified by the relations:

$$u[n]*h_d[n] = [u(t)*h_c(t)]\big|_{t=nT} \text{ and } H_d(z) = (1-z^{-1})Z\{[u(t)*h_c(t)]\big|_{t=nT}\}.$$

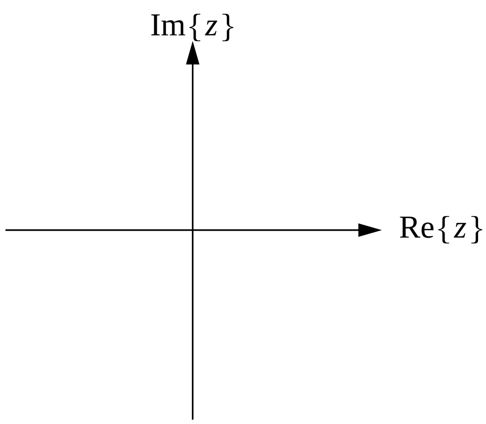
(b) (5 pts.) Consider the causal, first-order CT system with transfer function:

$$H_c(s) = \frac{a}{s+a}.$$

Sketch the poles and zeros of $H_c(s)$ in the *s*-plane, and state the condition on *a* for this system to be stable.



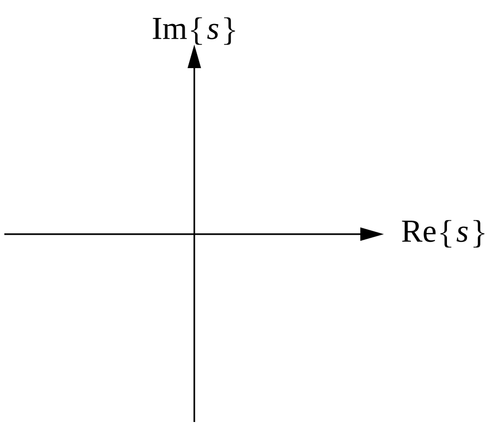
(c) (15 pts.) Using the results of part (a), find the step-invariant $H_d(z)$ for the $H_c(s)$ given in part (b). Sketch the poles and zeros of $H_d(z)$ on the z-plane as a function of a and T, and state the conditions on a and T for the system described by $H_d(z)$ to be stable. You may find it useful to define $\alpha = e^{-aT}$.



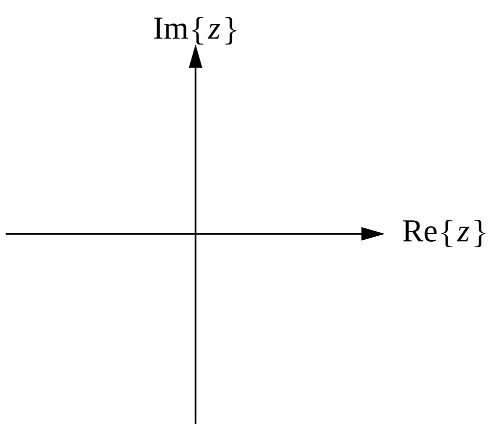
(d) (5 pts.) Consider the causal, second-order CT system with transfer function:

$$H_c(s) = \frac{s}{s^2 + 2s + 101}.$$

Sketch the poles and zeros of $H_c(s)$ in the s-plane, and state whether the system is stable. What type of system is this (e.g., integrator, highpass filter, etc.)?

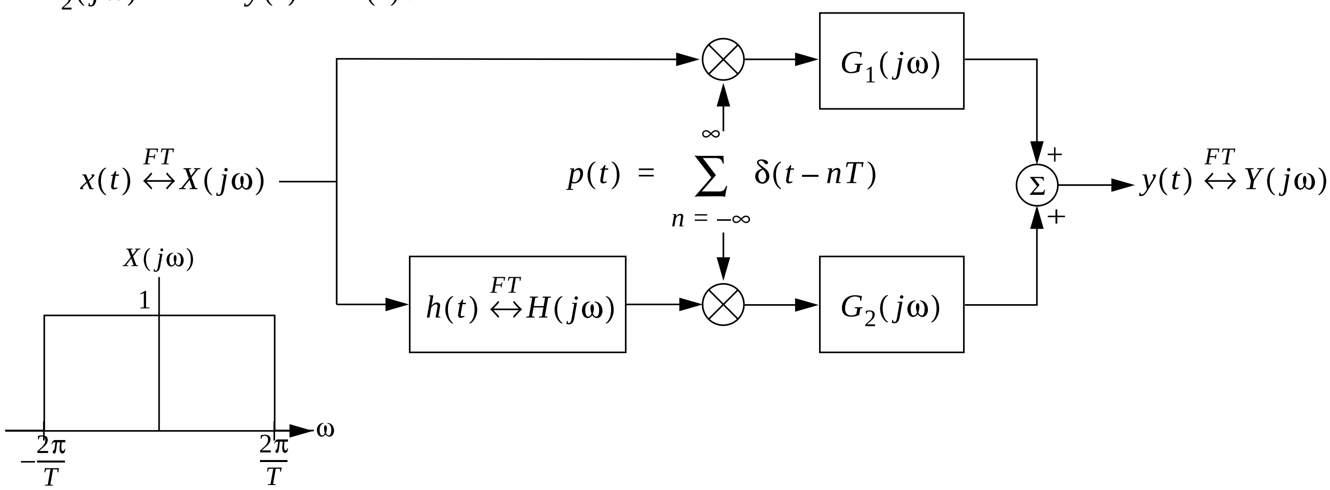


(e) (15 pts.) Suppose $T = 0.02$ s. Using the results of part (a), find the step-invariant $H_d(z)$ for the $H_c(s)$ given in part (d). Sketch the poles and zeros of $H_d(z)$ on the z-plane, and state if the system described by $H_d(z)$ is stable. You may want to define $r = e^{-T} \approx 0.98$ and $\theta = 10T = 0.2 \text{ rad} \approx 11°$.



Problem 5 (35 pts.) Let a CT LTI system be described by $h(t) \overset{FT}{\leftrightarrow} H(j\omega)$. Suppose that $H(j\omega)$ is periodic in ω, i.e., $H(j(\omega + \omega_0)) = H(j\omega), \forall\omega$, for some ω_0. Describe $h(t)$ as precisely as you can, and show how to express $h(t)$ in terms of an integral of $H(j\omega)$ over one period, e.g., $0 \le \omega < \omega_0$. *Hint*: it might help to first think about some specific example(s).

Problem 6 (40 pts.) This problem concerns sampling and reconstruction, i.e., conversion of CT signals to DT and back again. To simplify the mathematics, we will consider an equivalent all-CT system. The signal $x(t) \overset{FT}{\leftrightarrow} X(j\omega)$ is bandlimited to $|\omega| < 2\pi/T$. Samples are taken of $x(t)$ and of $x(t)*h(t)$ at a rate of 1/*T*, half the rate that would be required to specify $x(t)$ if only $x(t)$ were sampled. This problem shows that for some $h(t) \overset{FT}{\leftrightarrow} H(j\omega)$, it is possible to find $G_1(j\omega)$ and $G_2(j\omega)$ so that $y(t) = x(t)$.



(a) (15 pts.) Find an expression for $Y(j\omega)$.

(b) (5 pts.) Find a simpler expression for $Y(j\omega)$ that is valid for $-2\pi/T \le \omega \le 2\pi/T$, i.e., including only those terms that contribute over that frequency range.

(c) (20 pts.) Show that if $x(t)*h(t) = dx/dt$ and if $G_1(j\omega)$ and $G_2(j\omega)$ are as specified, then $y(t) = x(t)$. *Hint*: it is probably easiest to prove this using a series of carefully labeled sketches. *Please be honest in answering the question. If you make an error in some intermediate step but miraculously obtain the desired answer, you will receive no credit.*