EECS 120 Signals & Systems — University of California, Berkeley: Spring 2005

Ramchandran — March 16, 2005

Midterm Exam

Last name	First name	SID

Rules.

- You have two hours to complete this exam.
- There are 100 points for this exam. Points for the individual problems and subproblems are marked in the problem statement.
- The exam is closed-book and closed-notes; calculators, computing and communication devices are *not* permitted.
- However, one handwritten and *not photocopied* double-sided sheet of notes is allowed.
- Moreover, you receive, together with the exam paper, copies of Tables 4.2 and 5.2 of the course textbook.
- No form of collaboration between the students is allowed. If you are caught cheating, you may fail the course and face disciplinary consequences.

Please read the following remarks carefully.

- Show all work to get any partial credit.
- Take into account the points that may be earned for each problem when splitting your time between the problems.

Problem	Points earned	out of
Problem 1		40
Problem 2		15
Problem 3		20
Problem 4		25
Total		100

Problem 1 (*Short questions.)* 40 Points (5 Points each)

(a) For the following system with input x[n] and output y[n], circle whether the statements are true or false.

$$y[n] = \frac{1}{1 + x[2n]}$$

T F the system is linear
T F the system is time-invariant
T F the system is memoryless
T F the system is stable
T F the system is causal

(b) A discrete-time system has input x(t) and output y(t) such that

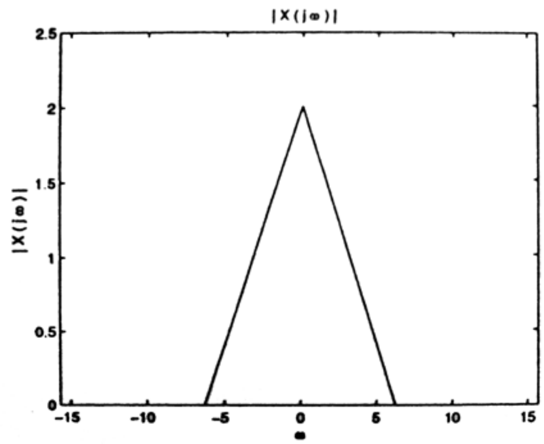
$$y[n] = x[n] – y[n – 1]$$

Circle whether the system is stable or unstable. Give an explanation in the additional box. If it is stable, give a short proof. If not, give a counterexample.

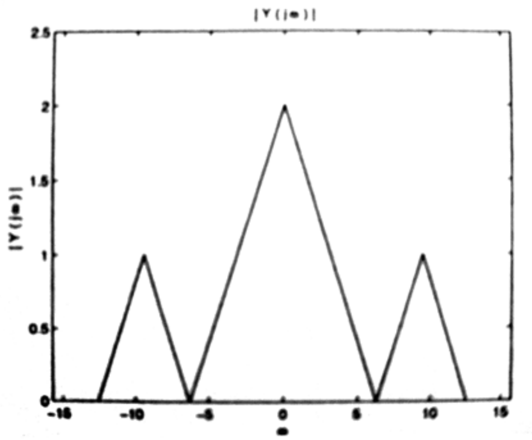
The system is:	stable unstable

(c) A signal x(t) is the input to an unknown system. The signal y(t) is the output. The magnitudes of the spectra of the input and output signals are given below.

|X(jw)|



|Y(jw)|



True, or False, or Not Enough Information

T	F	NEI	The system is both linear and time-invariant

Explain your answer, briefly (approx 1-3 sentences).

(d)

$$\text{Given } x(t) = \begin{cases} 1, & 0 \le t < \tfrac{1}{2}, \\ -1, & \tfrac{1}{2} \le t < 1, \\ 0, & \text{otherwise} \end{cases}$$

Plot x(t/4 – 3). Label your axes clearly and carefully!

(e) A discrete-time LTI system with input x[n] and output y[n] is described by the following constant coefficient difference equation.

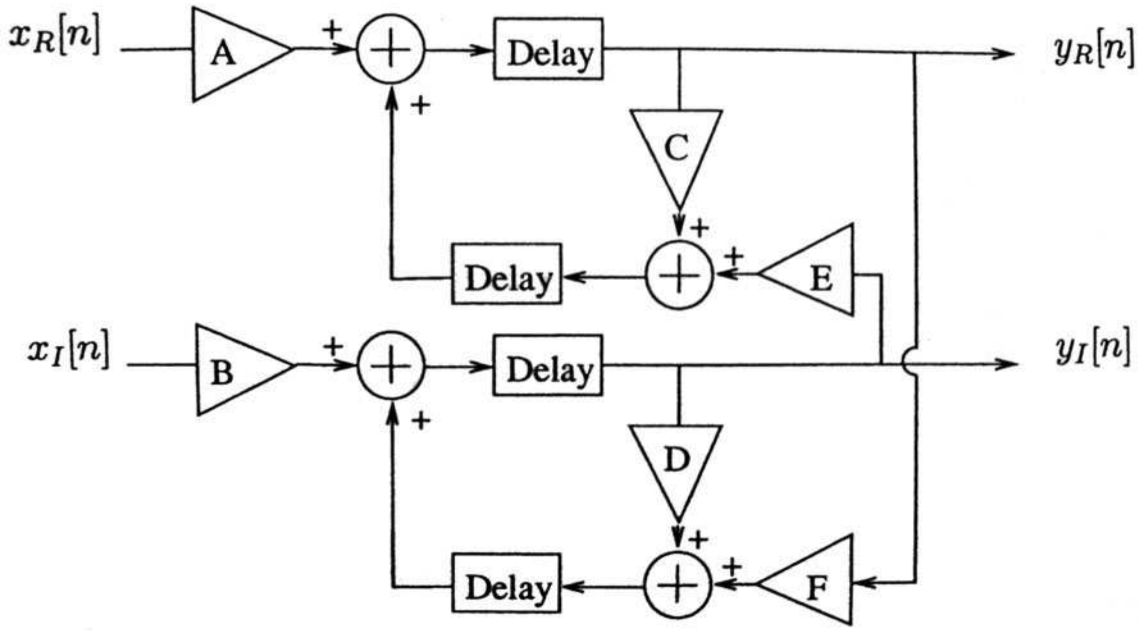
$$y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = x[n]$$

If x[n] = cos(πn), what is y[n]?

y[n] =

(f) Find the correct real gains in the block diagram below so that its input and output are related by the difference equation:

$$y[n] - \frac{1}{4} e^{j\frac{\pi}{3}} y[n-2] = \frac{1}{3} x[n-1]$$



A =	C =	E =
B =	D =	F =

(g) The signals $x_1(t)$ and $x_2(t)$ are defined below.

$$x_1(t) = \begin{cases} 1 - |t|, & |t| < 1 \\ 0, & |t| \geq 1 \end{cases}$$

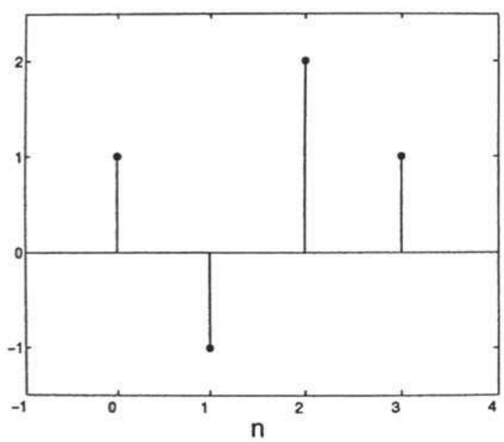
$$x_2(t) = \delta(t + 2) + 2\delta(t-2)$$

Plot the convolution of the two signals, $y(t) = x_1(t) * x_2(t)$, clearly labeling the time axis and amplitudes.

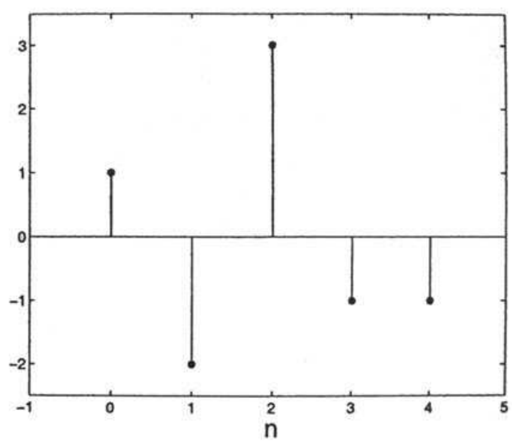
(h) A given discrete-time LTI system has impulse response $h[n]$, input $x[n]$, and output $y[n]$. $h[n]$ and $y[n]$ are given below.

$$h[n] = \delta[n] - \delta[n-1] + 2\delta[n-2] + \delta[n-3]$$
$$y[n] = \delta[n] - 2\delta[n-1] + 3\delta[n-2] - \delta[n-3] - \delta[n-4]$$

h[n]



y[n]



Given that $x[n]$ is causal, graph $x[n]$, for $-1 \leq n \leq 5$, carefully labeling the time axis and the amplitudes.

Problem 2 15 Points

(a) (3 Points)

We are given the following information about a signal x(t).

1. x(t) has period 2π
2. x(t) has a Fourier series expansion with coefficients a_k.
3. $a_k = 0$ if $|k| > 2$.

Write down the Fourier Series expansion of x(t), simplifying as much as possible.

(b) (6 points)

We are given more information about x(t).

4. x(t) is real and off.
5. $x(t - \pi) = - x(t)$

What does this say about a_0 and a_2?

$a_0 =$	$a_2 =$

(c) (3 Points)

The final fact about x(t) given is:

6. $\frac{1}{2\pi}\int_0^{2\pi} |x(t)|^2 \, dt = 2$

What does this say about a_1 ?

(d) (3 Points)

Graph x(t) for t in $[0,2\pi]$. Carefully label the time axis and amplitudes.

Problem 3 20 Points

(a) (i) (5 Points)

Find the Fourier transform X(jw) of

$$x(t) = \begin{cases} 4 - |t|, & |t| \leq 4 \\ 0, & \text{otherwise.} \end{cases}$$

(ii)

The signal y(t) is defined below

$$y(t) = \begin{cases} t + 8, & -8 \leq t < -4, \\ 4, & -4 \leq t < 4, \\ 8 - t, & 4 \leq t < 8, \\ 0, & \text{otherwise.} \end{cases}$$

Find the Fourier transform Y(jw) of y(t).

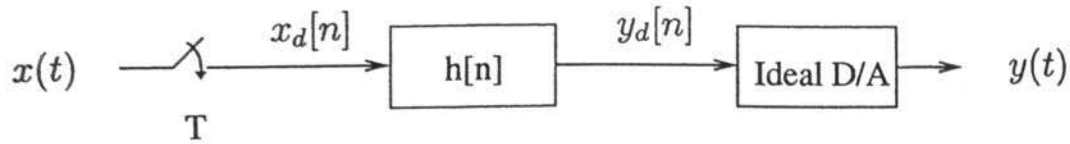
(b) (7 Points)

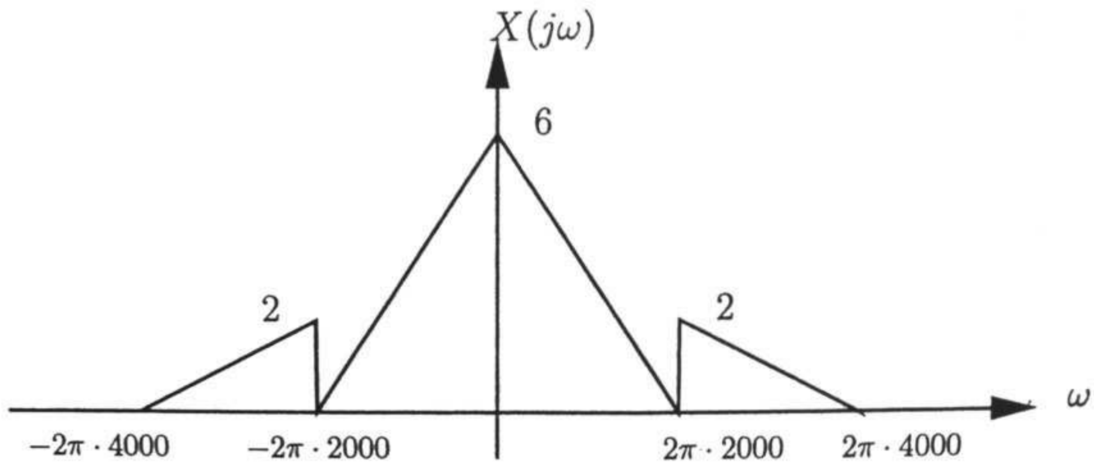
Compute the following integral:

$$\int_{-\infty}^{\infty}\left(\frac{\sin(7\tau)}{\pi\tau}\right)\left(\frac{\sin(3(\pi/4-\tau))}{\pi(\pi/4-\tau)}\right)d\tau$$

$$\int_{-\infty}^{\infty}\left(\frac{\sin(7\tau)}{\pi\tau}\right)\left(\frac{\sin(3(\pi/4-\tau))}{\pi(\pi/4-\tau)}\right)d\tau =$$

Problem 4 25 Points





A sampler and a filter implemented in discrete time are shown in the diagram above. The signal x(t) has spectrum X(jw) shown above.

In the spectrum of x(t), the content in the range of $|w| \in [4000\pi, 8000\pi]$ is considered to be "noise". The content in the range of $|w| < 4000\pi$ is the signal of interest.

(a) (5 Points)

What is the Nyquist sampling rate so that the sampler avoids aliasing in $x_d[n]$?

$T_{nyquist} =$

(b) (5 Points)

Our goal is to recover the signal of interest in y(t), and hence we want to unaliased version of the signal of interest in $x_d[n]$.

What is the maximum value of T which avoids aliasing the signal of interest?

T_{max} =

(c) (5 Points)

Now fix T = 1/7000 seconds. Plot $X_d(e^{jw})$, the spectrum of $x_d[n]$, clearly labeling the frequency axis and amplitudes.

(d) (5 Points)

Assuming that T = 1/7000 seconds, let $h_d[n]$ be the filter you need to completely eliminate all the noise while keeping your signal intact. Plot $H_d(e^{jw})$, the spectrum of $h_d[n]$, clearly labeling the frequency axis and amplitudes.

(e) (5 Points)

Write a closed form expression for y(t) in terms of $y_d[n]$.

y(t) =