

LAST Name _____ FIRST Name _____

Lab Time _____

- **(10 Points)** Print your name and lab time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 90 minutes to complete. You will be given at least 90 minutes, up to a maximum of 110 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except three double-sided 8.5" × 11" sheets of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—*including, for example, commencing work prematurely or continuing beyond the announced stop time*—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 16.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the sixteen numbered pages. If you find a defect in your copy, notify the staff immediately.
- **You will be given a separate document containing formulas and tables.**
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

MT2.1 (30 Points) A particular operational amplifier (op-amp) can be thought of as a causal and stable LTI filter $P : [\mathbb{R} \rightarrow \mathbb{R}] \rightarrow [\mathbb{R} \rightarrow \mathbb{R}]$. Let P and \hat{P} denote the frequency response and transfer function of the op-amp, respectively. Suppose the transfer function \hat{P} is given by:

$$\hat{P}(s) = \frac{G_0}{s + \omega_0},$$

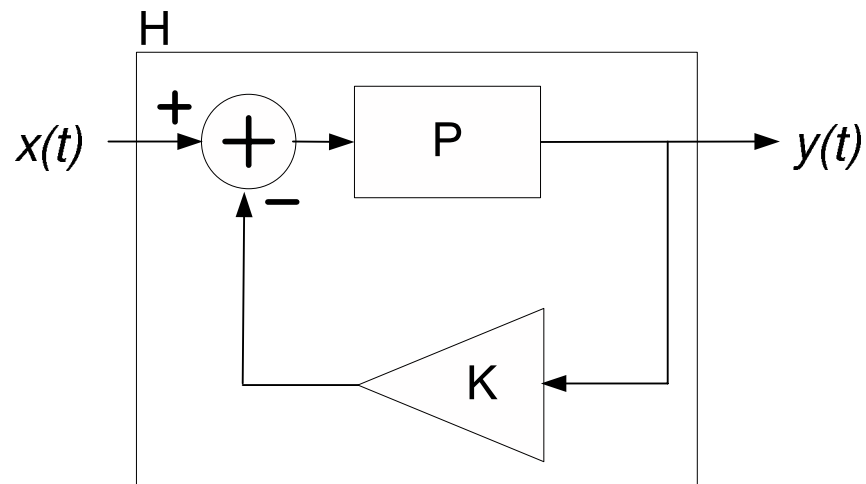
where $G_0 = 10^7$ and $\omega_0 = 50$ rad/sec (approximately 8 Hz). At one or more points in this problem, you may find it useful to know that $20 \log 2 \approx 6$ dB.

NOTE: Throughout this problem, you are encouraged to make *reasonable* numerical approximations whenever appropriate. However, please explicitly articulate any approximation you make, so we can track your work.

- (a) Provide a well-labeled Bode plot of $|P(\omega)|$, the magnitude of the frequency response P . Be sure to identify the plot's salient features, such as the flat gain (in dB) and every cutoff frequency, in terms of G_0 and ω_0 . Also determine these salient features numerically.

Is the op-amp a high-pass, band-pass, or low-pass filter?

- (b) The op-amp of part (a) has a high flat gain, but a narrow bandwidth. We wish to improve the bandwidth by placing the op-amp in a proportional feedback configuration H , as shown in the figure below, where K denotes a proportional constant:



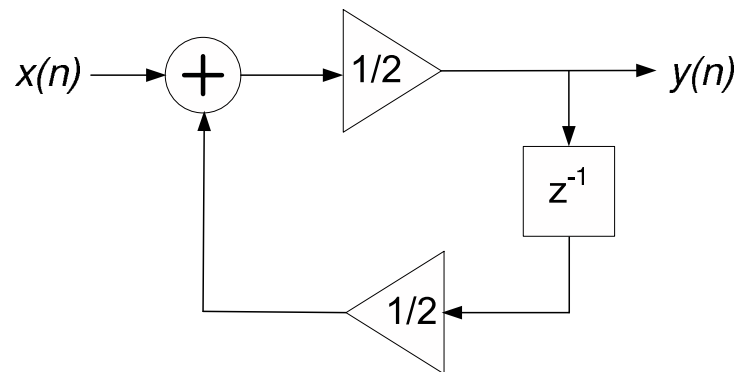
- (i) Determine the transfer function \hat{H} of the feedback system H , in terms of G_0 , ω_0 , and K .

(ii) Provide a well-labeled Bode plot of $|H(\omega)|$, the magnitude of the frequency response H of the system H . Be sure to identify the plot's salient features, such as the flat gain (in dB) and every cutoff frequency, in terms of G_0 , ω_0 , and K . Also determine these salient features numerically, for $K = 0.1$.

(iii) What is the engineering tradeoff between the two systems of parts (a) and (b)?

(iv) Is the feedback system H a high-pass, band-pass, or low-pass filter?

MT2.2 (40 Points) A causal LTI filter $F : [\mathbb{Z} \rightarrow \mathbb{R}] \rightarrow [\mathbb{Z} \rightarrow \mathbb{R}]$ has impulse response f , frequency response F , and transfer function \hat{F} . The filter is characterized, at least in part, by the following delay-adder-gain block diagram:



NOTE: You can approach parts (b)–(e) of this problem independently, and in any order. However, each of these parts depends on part (a).

(a) Determine \hat{F} and its corresponding region of convergence.

(b) Determine a simple expression for the impulse response f .

(c) (i) What is the linear, constant-coefficient difference equation governing the input-output behavior of the filter?

(ii) Suppose $y(-1) = 4$, and that the input signal x is causal, i.e., $x(n) = 0$, $n < 0$. Determine $y_{\text{ZIR}}(n)$, $n \geq 0$, the sample values of the zero-input response y_{ZIR} .

- (d) For the summation below, either determine a numerical value to which it converges, or explain why it does not converge.

$$\sum_{n=-\infty}^{\infty} e^{i\pi n} f(n).$$

- (e) (i) For the frequency range $-\pi < \omega \leq +\pi$, provide a well-labeled sketch of $|F(\omega)|$, the magnitude response of the filter F.
Is the filter F low-pass, band-pass, or high-pass?

(ii) Suppose the following input signal x is applied to the filter:

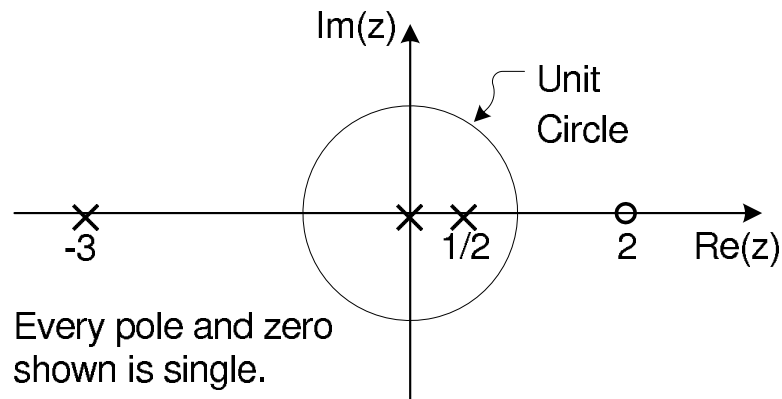
$$\forall n, \quad x(n) = \cos\left(\frac{\pi}{3}n\right) u(n).$$

Determine a simple expression for $y_{ss}(n)$, the sample values of the corresponding steady-state response y_{ss} .

Hint: Find the numerical value of $|F(\pi/3)|$. You may do this, for example, by employing the geometric method of measuring distances from the unit circle to each pole and zero.

In carrying out your work, you may find the following approximations useful: $2/\sqrt{13} \approx 0.55$; $\pi/3 \approx 1.04$; and $\tan^{-1} 2\sqrt{3} \approx 1.29$.

MT2.3 (40 Points) A *BIBO stable* LTI filter $F : [\mathbb{Z} \rightarrow \mathbb{R}] \rightarrow [\mathbb{Z} \rightarrow \mathbb{R}]$ has impulse response f , frequency response $F : \mathbb{R} \rightarrow \mathbb{C}$, and rational transfer function $\hat{F} : \mathbb{C} \rightarrow \mathbb{C}$. Every finite pole and zero of the filter is shown in the pole-zero diagram below and has multiplicity 1.



We also know that the DC gain of the filter is $-1/2$, i.e.,

$$F(\omega)|_{\omega=0} = -\frac{1}{2}.$$

NOTE: You can approach parts (b) and (c) of this problem independently, and in either order. However, each of these parts depends on part (a).

- (a) Determine $\hat{F}(z)$ and its corresponding region of convergence.

- (b) Consider an LTI filter $G : [\mathbb{Z} \rightarrow \mathbb{R}] \rightarrow [\mathbb{Z} \rightarrow \mathbb{R}]$ having impulse response g , where

$$\forall n \text{ and } \exists N, \quad g(n) = f(n + N).$$

The transfer function \widehat{G} of the filter G has exactly one zero at $z = 0$.

- (i) Determine N and provide a well-labeled pole-zero diagram for the transfer function \widehat{G} . Be sure to specify the multiplicity of every pole and zero, whether finite or infinite.

- (ii) Explain whether G is a causal filter, and determine the numerical value of $g(0)$.

- (c) Consider an LTI filter $\bar{F} : [\mathbb{Z} \rightarrow \mathbb{R}] \rightarrow [\mathbb{Z} \rightarrow \mathbb{R}]$ whose impulse response \bar{f} is obtained from f by "time reversal", i.e.,

$$\forall n, \quad \bar{f}(n) = f(-n).$$

- (i) Provide a pole-zero diagram for the transfer function $\widehat{\bar{F}}$ and determine its corresponding region convergence. Be sure to specify the location and multiplicity of *every* pole and zero, whether finite or infinite.

- (ii) Without much algebraic manipulation, determine an expression for $\widehat{\bar{F}}(z)$.

- (iii) Let the LTI filter $H : [\mathbb{Z} \rightarrow \mathbb{R}] \rightarrow [\mathbb{Z} \rightarrow \mathbb{R}]$ have an impulse response h characterized as follows:

$$\forall n, \quad h(n) = (f * \bar{f})(n).$$

Determine \hat{H} , the transfer function of the filter H . What is the corresponding region of convergence?

You may use the blank space below for scratch work. Nothing written below this line on this page will be considered in evaluating your work.

MT2.4 (40 Points) Consider an FIR filter $H : [\mathbb{Z} \rightarrow \mathbb{R}] \rightarrow [\mathbb{Z} \rightarrow \mathbb{R}]$ having impulse response h , frequency response H , and transfer function \hat{H} .

Each part below discloses partial information about the filter H . Ultimately, your task is to determine the impulse response h completely.

In the space provided for each part, state every inference that you can draw from the information disclosed up to, and including, that part.

Justify all your work succinctly, but clearly and convincingly.

(I) $\hat{H}(z)$ has *at least* one pole at $z = 0$.

(II) $\hat{H}(1/z)$ has *no* pole at $z = 0$. If you synthesize this fact properly, you will *not* need the information presented in part (I); there is some redundancy built into these first two facts.

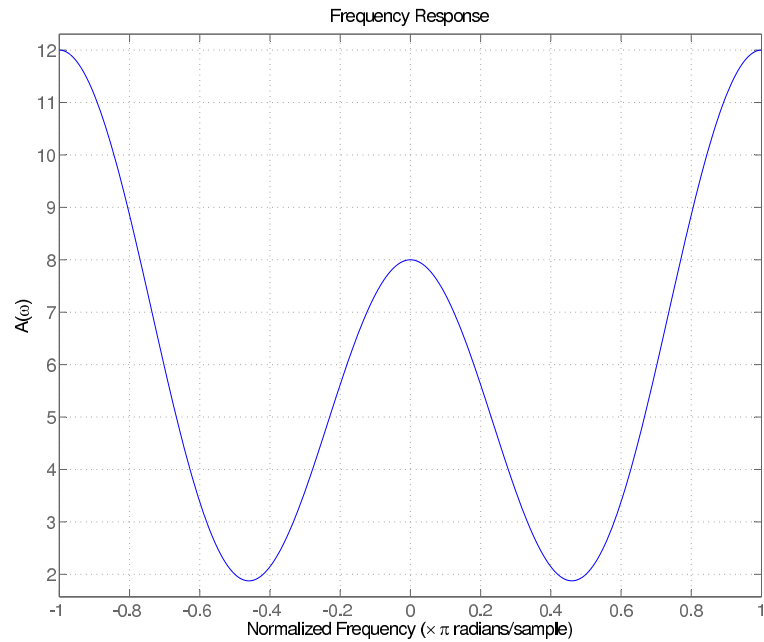
(III) There exists a filter $A : [\mathbb{Z} \rightarrow \mathbb{R}] \rightarrow [\mathbb{Z} \rightarrow \mathbb{R}]$ having impulse response a and frequency response A , about which we know the following:

(i) $\forall \omega \in \mathbb{R}, A(\omega) = H(\omega) e^{i2\omega}$.

(ii) $\int_{\langle 2\pi \rangle} A(\omega) d\omega = 12\pi$, where $\langle 2\pi \rangle = [0, 2\pi], [-\pi, \pi]$, or another continuous interval of length 2π .

(iii) $\forall \omega \in \mathbb{R}, A(\omega) \in \mathbb{R}$.

(iv) The figure below depicts $A(\omega), \forall \omega \in [-\pi, +\pi]$.



Determine, and provide a well-labeled plot of, the impulse response h .

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Problem Name	Points	Your Score
1	30	
2	40	
3	40	
4	40	
Total	160	