EE 121, Spring 2001
Midterm 1
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Problem #1

1.

$S_u(f) = |H(f)|^2 * S_{nw}(f) = N_0A^2/2 * \pi(f / 2w)$

$v(t) = u(t)sin(2\pi f_0 t)$

$R_v(t+\tau, t) = E[v(t+\tau)\ v(t)]$

$= E[u(t+\tau)\ u(t)\ sin(2\pi f_0\ (t+\tau))\ sin(2\pi f_0\ t)]$

$= 0.5\ E[u(t+\tau)\ u(t)\ (cos(2\pi f_0\ \tau) - cos(4\pi f_0\ t+2\pi f_0\ \tau))]$

$= 0.5\ R_u(\tau)\ cos(2\pi f_0\ \tau) - 0.5\ R_u(\tau)\ cos(4\pi f_0\ t+2\pi f_0\ \tau)$

$$\frac{1}{T_0}\int_{t'}^{t'+T_0} Rv(t+\tau, t)dt' = Ru(\tau)\cos(2\pi * f_0 * \tau)$$

$S_v(f) = S_u(f) * [0.5\delta(f- f_0) + 0.5\delta(f+ f_0)]$

$= 0.5\ S_u(f- f_0) + 0.5\ S_u(f+ f_0)$

$= N_0A^2/8 * \pi((f- f_0) / 2w) + N_0A^2/8 * \pi((f+ f_0) / 2w)$

$S_w(f) = |H(f)|^2 * S_v(f)$

$= N_0A^4/8 * \pi(f / 2w) * \pi((f- f_0) / 2w) + N_0A^4/8 * \pi(f / 2w) * \pi((f+ f_0) / 2w)$

Problem #2

2.

$V(f) = M(f) * [1/2j * \delta(f - f_1) - 1/2j * \delta(f + f_1)]$

$= 1/2j * M(f - f_1) - 1/2j * M(f + f_1)$

Since $f_1 >> W$ the Fourier Transform of the corresponding analytic signal is

$Z(f) = 1/j * M(f - f_1)$

$z(t) = 1/j * m(t)\ e^{\wedge}(j*2\pi f_1\ t)$

$u_l(t) = 1/j * m(t)\ e^{\wedge}(j*2\pi(f_1 - f_0)t)$

$= m(t)\ sin\ (2\pi(f_1 - f_0)t) - jm(t)cos(2\pi(f_1 - f_0)t)$

Hence

$u_c(t) = m(t)\ sin\ (2\pi(f_1 - f_0)t)$

$u_s(t) = -m(t)\ cos\ (2\pi(f_1 - f_0)t)$

Note: It is irrelevant whether $f_0 >> W$ or that $| f_1 - f_0 |$ is small relative to f_1 and f_0

Problem #3

3.

S(t) is even, so

$$s(t) = s_0/2 + \sum_{n=1}^{\infty} s_n \cos(2\pi * n * f_c * t)$$

for some $s_0, s_1, s_2 \ldots$

$$u(t) = m(t)\, s(t) = s_0/2 * m(t) + \sum_{n=1}^{\infty} s_n m(t) \cos(2\pi * n * f_c * t)$$

$v(t) = s_1 m(t) \cos(2\pi * f_c * t)$ *because* $f_c >> W$

$n(t)$ *is bandpass noise with flat power spectral density* $N_0/2$ *over the bands* $\pm f_c \pm W$

$r(t) = s_1 m(t) \cos(2\pi * f_c * t) + n_c(t) \cos(2\pi * f_c * t) - n_s(t) \sin(2\pi * f_c * t)$

$r(t) \cos(2\pi * f_c * t)$ *after low pass filtering yields* $1/2 * [s_1 m(t) + n_c(t)]$

The message signal power at the output is

$$P_0 = 1/4 * s_1^2 * P_M$$

The noise power at the output is

$P_{n0} = 1/4 * P_{nc} = 1/4 * N_0/2 * 4W = N_0 W/2$

$(S/N)_0 = P_0/P_{n0} = s_1^2 P_M / 2WN_0$

Here $s_1 = 2/T_c \int_{-T_c/4}^{T_c/4} A_c \cos(2\pi f_c * t)\,dt$

$- \int_{-T_c/2}^{T_c/4} A_c \cos(2\pi f_c * t)\,dt$

$- \int_{-T_c/4}^{T_c/2} A_c \cos(2\pi f_c * t)\,dt$

$= 4/\pi * A_c$

The reserved power is

$P_R = \sum_{n=1}^{\infty} s_n^2 P_M$ *because* $f_c >> W$ *and because* $s_0 = 0$

By Parseval's relation, since $s_0 = 0$

$$\sum_{n=1}^{\infty} s_n^2 = A_c^2$$

Thus

$$P_R = A_c^2 * P_M$$

Hence $(S/N)_0 = (16/\pi^2) * (A_c^2 * P_M / 2WN_0) = 8/\pi^2 * P_R/WN_0$

$= 8/\pi^2 * (S/N)_b$

Problem #4

4.

Let $x_1 < x_2$ be the quantization levels chose. Letus write $u = (x_1+x_2)/2$ and

$x_1 = u - v$

$x_2 = u+v$

Given u, the choice of v is decide by

(A) $\int_{-\infty}^{u} x\phi(x)dx = (u-v) \int_{-\infty}^{u} \phi(x)dx$

(B) $\int_{u}^{\infty} x\phi(x)dx = (u+v) \int_{u}^{\infty} \phi(x)dx$

where $\rho(n)$ derives the Gaussian density

$(1/(\sqrt{2\pi} * \sigma)) * e^{-x^2/2\sigma^2}$

The mean square distribution is

$$\int_{-\infty}^{u} (x-(u-v))^2 \phi(x)dx + \int_{u}^{-\infty} (x-(u+v))^2 \phi(x)dx$$

$$= \int_{-\infty}^{u} x^2\phi(x)dx - (u-v)^2 \int_{-\infty}^{u} \phi(x)dx + \int_{u}^{\infty} x^2\phi(x)dx - (u+v)^2 \int_{u}^{\infty} \phi(x)dx$$

where we used (A) and (B)

using (A) and (B) again, this can be written as

$$= \sigma^2 - ((\int_{-\infty}^{u} x\phi(x)dx)^2 + (\int_{u}^{\infty} x\phi(x)dx)^2)$$

so we want to choose u to maximize

$$(\int_{-\infty}^{u} x\phi(x)dx)^2 + (\int_{u}^{\infty} x\phi(x)dx)^2$$

clearly the best choice is $u=0$

Then v would be chosen so that $u-v = x_1$ is the centroid of the left half and $u+v = x_2$ is the centroid of the right half of the density

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