EECS 123 Digital Signal Processing
Gastpar

University of California, Berkeley: Fall 2007
November 7, 2007

Exam 2

- If we can't read it, we can't grade it.
- We can only give partial credit if you write out your derivations and reasoning in detail.
- You may use the back of the pages of the exam if you need more space.

*** GOOD LUCK! ***

Problem	Points earned	out of
Problem 1	20	20
Problem 2	20	20
Problem 3	11	20
Problem 4	10	25
Problem 5	0	15
Total		100

Partial Credit.

Partial credit will be given only if there is sufficient information in your work. In general, a good way to show that you understand what's going on is for example to provide plots, sketches, and formulas for *intermediate* signals. That way, if you make an error somewhere along the way, we can trace it and evaluate whether or not you understood the basics of the problem.

Useful Formulae.

- For the continuous-time box function,

$$b(t) = \begin{cases} 1, & -T \leq t \leq T \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

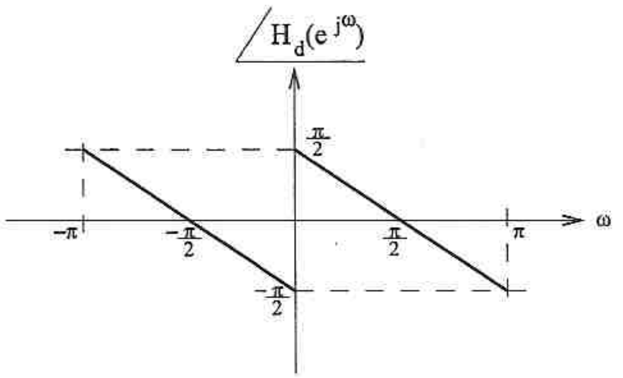
  the (continuous-time) Fourier transform is given by

$$B(j\Omega) = \frac{2\sin(\Omega T)}{\Omega}. \tag{2}$$

- $\tan(\pi/4) = 1$

Problem 1 *(Phase Properties.)* 20 Points

Given the phase characteristics of a generalized linear phase FIR filter $H_d(e^{j\omega})$ shown below, answer the following questions. Include brief explanations to get credit.



(a) (6 pts) Is this a symmetric (i.e., Type-I or Type-II) or an anti-symmetric (i.e., Type-III or Type-IV) filter? Why?

$\text{Slope} = \frac{-\pi/2}{\pi/2} = -1 \rightarrow \alpha = 1 \rightarrow M = 2$ (even)

$\beta = \pi/2$ and M even $\rightarrow$ $\boxed{\text{Type-III (anti-symmetric) filter}}$

6/6

(b) (8 pts) Can the filter length be determined from the given information? If yes, what is the length? If not, why not?

$M = 2\alpha = 2 \cdot 1 = 2$

length $= M+1 = \boxed{3}$

8/8

(c) (6 pts) The filter magnitude response has a DC gain of 1. True or False? Why?

$|H(0)| = 1 \rightarrow |h(0) + \cancel{h(1)} + h(2)| = |h(0) + (-h(0))| = 0$

$h(1) = -h(1)$, so $h(1) = 0$

antisymmetric

but

$H(0) = h(0) + h(1) + h(2)$

$0 \neq 1.$ $\boxed{\text{False}}$

6/6

$T_d^2 + \cancel{2T_d} + 1$
$+ 1 - \cancel{2T_d} + T_d^2$
$2(T_d^2 + 1)$

Problem 2 *(Filter Design.)* 20 Points

(a) (12 pts) A continuous-time filter is given by $H_a(s) = \frac{2}{2+s}$. We want to use this as a prototype filter to design a discrete-time filter *via the bilinear transform* with a suppression of $1/\sqrt{2}$ at $\omega_c = \pi/2$. (Note that the filter has a gain of 1 at frequency zero.) Give $H(z)$ explicitly.

12/12

$$H_a(s) = \frac{2}{2+s} \qquad \omega_c = \pi/2 \rightarrow z = i$$

$$H_a(z) = H_a(s)\Big|_{s = \frac{2}{T_d}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)} = \frac{2}{2 + \frac{2}{T_d}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$H_a(z) = \frac{T_d(1+z^{-1})}{T_d(1+z^{-1}) + (1-z^{-1})} \rightarrow H_a(i) = \frac{1}{\sqrt{2}} \rightarrow \frac{T_d(1-i)}{T_d(1-i) + (1+i)}$$

$$H_a(i) = \frac{T_d - T_d i}{(T_d+1) + (1-T_d)i}$$

$$|H_a(i)| = \frac{T_d\sqrt{2}}{\sqrt{T_d^2+1}\,\sqrt{2}} = \frac{1}{\sqrt{2}}$$

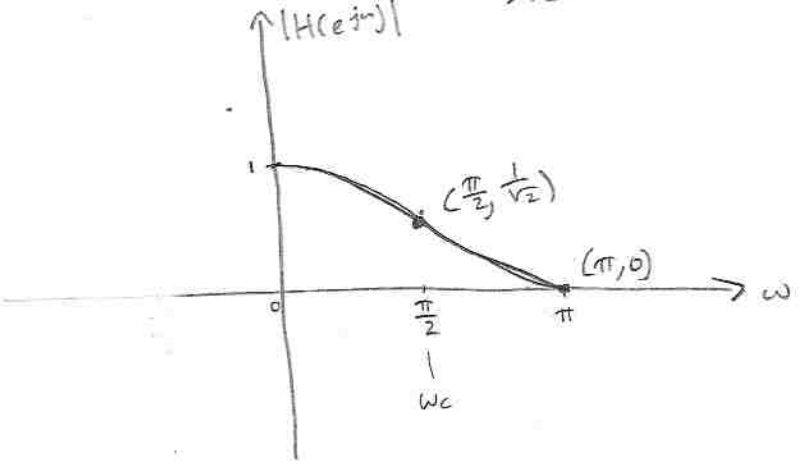
$$\sqrt{2}\,T_d = \sqrt{T_d^2+1} \rightarrow 2T_d^2 = T_d^2 + 1$$
$$T_d^2 = 1 \rightarrow T_d = 1$$

$$\boxed{H_a(z) = \frac{1+z^{-1}}{2}}$$

(b) (8 pts) Sketch $|H(e^{j\omega})|$ for the filter designed in part (a) over the interval $0 \leq \omega \leq \pi$.

$$H_a(z) = \frac{z+1}{2z} \rightarrow H(e^{j\omega}) = \frac{e^{j\omega}+1}{2e^{j\omega}} \rightarrow |H(e^{j\omega})| = \frac{|1+e^{j\omega}|}{2}$$

8/8



$1 + e^{j2\omega}$

Problem 3 *(Filter Design.)* 20 Points

(a) (5 pts) We want to approximate the lowpass filter in Figure 1 with the optimal minimax (Parks-McClellan) Type-I filter $h(n)$. Just like in class, the band $0 \le \omega \le \omega_p$ is the desired passband, and the band $\omega_s \le \omega \le \pi$ is the desired stopband. In Figure 1, provide a sketch the form of $H(e^{j\omega})$ when $h(n)$ has length 3. Recall that the amplitude of a Type-I filter has the form $A(e^{j\omega}) = \sum_{k=0}^{M/2} a_k \cos(k\omega)$.

$M/2 = 1$ length $3 \rightarrow M = 2 \rightarrow$ 1st order polynomial of $x = \cos\omega$; 0 alternations within F

5/5

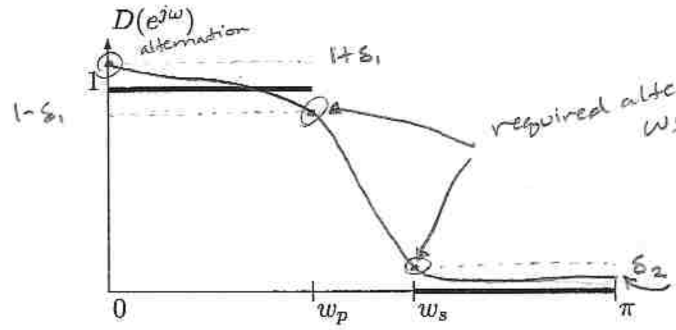


Figure 1:

(b) (10 pts) Determine the filter $h(n)$. *Hint:* If you find it easier, you may start by assuming that $\omega_p = \pi/3$ and thus, $\omega_s = 2\pi/3$.

$h(0) = h(2)$, $h(1) = 0$ Type-II symmetry

$h(1) = 0 \rightarrow a_1 = 0$

P-M $\rightarrow$ guess: $\underbrace{\omega_1 = 0,\ \omega_2 = \omega_p = \frac{\pi}{3},\ \omega_3 = \omega_s = \frac{2\pi}{3}}$

$$\begin{pmatrix} 1 & \cos(0) & -K \\ 1 & \cos(\pi/3) & K \\ 1 & \cos(2\pi/3) & -1 \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \\ \delta \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & -K \\ 1 & 1/2 & K \\ 1 & -1/2 & -1 \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \\ \delta \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$a_0 + K\delta = 1 \rightarrow$

$a_0 = -\delta$ $\Rightarrow$ next, find δ, compare w/ new δ, try new ω_i's, until minimax is achieved

$$\begin{pmatrix} 1 & 1 & K \\ 1 & -1/2 & K \\ 1 & -1/2 & 1 \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \\ \delta \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

Parks-McClellan

$\boxed{h(1) = 0}$

$\boxed{h(0) = h(2)}$

6/10

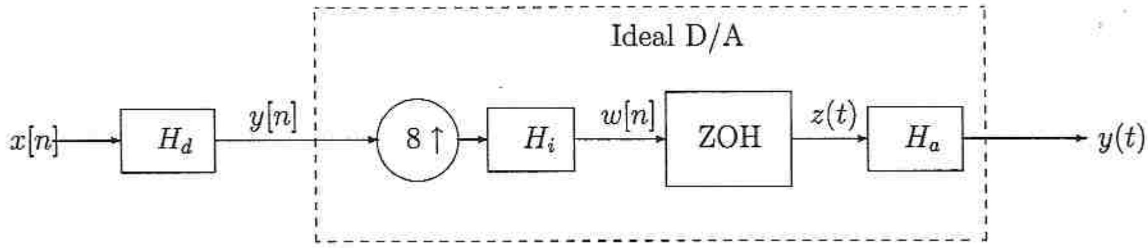
(c) (5 pts) What is the largest value of ω_p such that the maximum error is $\delta \leq 1/6$?

$\omega_p = \frac{\pi}{2}$

$E(\omega) = W(\omega)[H_d(e^{j\omega}) - A_e(e^{j\omega})]$

0/5

Problem 4 *(Multirate System.)* 25 Points



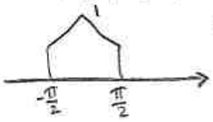
$$H_d(e^{j\omega}) = \begin{cases} 1, & |\omega| \le \frac{\pi}{2} \\ 0, & \text{else} \end{cases} \qquad H_i(e^{j\omega}) = \begin{cases} 8, & |\omega| \le \frac{\pi}{8} \\ 0, & \text{else} \end{cases}$$

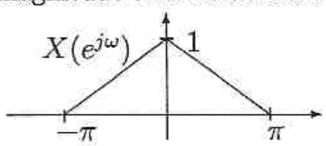
The ZOH operates at interval T but produces pulses of width $\frac{T}{4}$, i.e.

$$g(t) = \begin{cases} 1, & 0 \le t \le \frac{T}{4} \\ 0, & \text{else,} \end{cases}$$

and the output is $z(t) = \sum_{n=-\infty}^{\infty} w[n]g(t - nT)$.

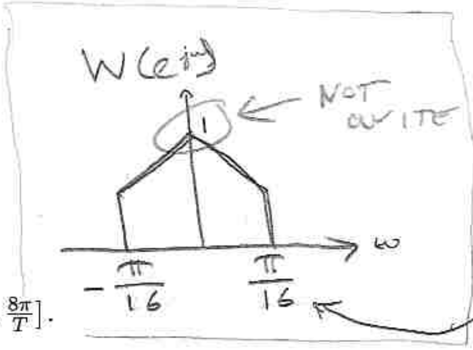
(a) (9 pts) For $X(e^{j\omega})$ pictured, sketch $W(e^{j\omega})$. Label the magnitude and bandwidth.





6/9

$8\uparrow \sim y(t) = x(\frac{n}{8}) = \sum_n x(n)\,\delta(t-nT)$



$\frac{\pi}{2L}$

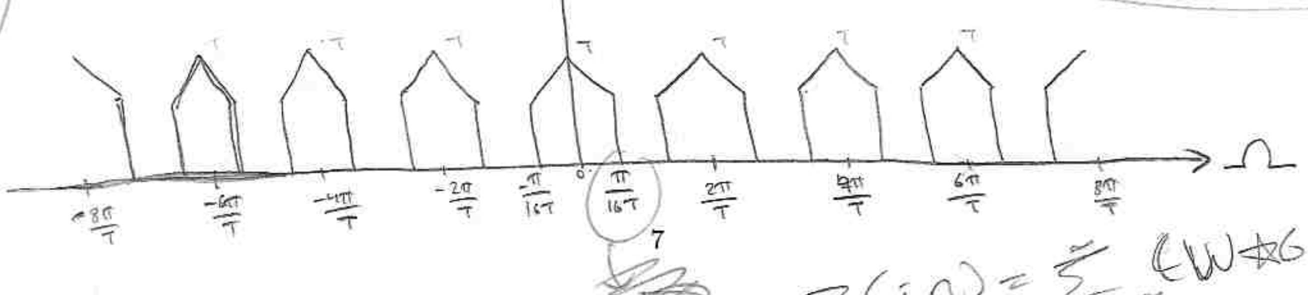
(b) (7 pts) For the same $X(e^{j\omega})$, sketch $|Z(j\Omega)|$ for $\Omega = [\frac{-8\pi}{T}, \frac{8\pi}{T}]$.

$Z(t) = \sum_{n=-\infty}^{\infty} w(n)\, g(t-nT)$

$Z(j\Omega) = \sum_{n=-\infty}^{\infty} W(j(\Omega - \frac{2\pi n}{T}))$

Should be convolution of W and G, actually

4/7

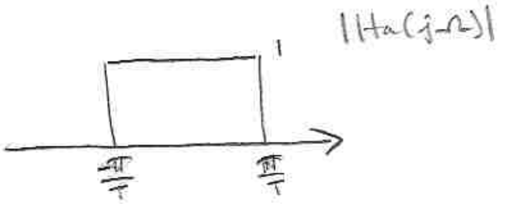


$Z(j\Omega) = \sum_{n=-\infty}^{\infty} (W * G)(j\Omega)$

(c) (7 pts) For this part, we are interested in making the dashed block (with input $y[n]$ and output $y(t)$) an ideal D/A converter for arbitrary $y[n]$, i.e. ignore the effects of H_d.

What is the "cheapest"[1] filter $H_a(j\Omega)$ such that between $y[n]$ and $y(t)$ we have an ideal D/A. Sketch $|H_a(j\Omega)|$ and specify its value where necessary.

transition band ~ main lobe width



0/7

$$z(t) = \sum_{n=-\infty}^{\infty} w(n)\, g(t-nT)$$

(d) (2 pts) Explain (in words) the advantages and disadvantages of this D/A converter design over a direct implementation (as we have discussed it in class).

0/2

This D/A converter design is cheaper than the direct implementation but it is not as exact. (due to the need to arrive at a cheap solution) accurate

The ZOH approximation will not produce the same results as higher-order holds or the sinc kernel reconstruction. HMM...

[1]the filter having the largest transition band (as we have seen in class, the smaller (i.e., steeper) the transition band, the more filter coefficients are necessary)

Problem 5 *(Filter Bank.)* 15 Points

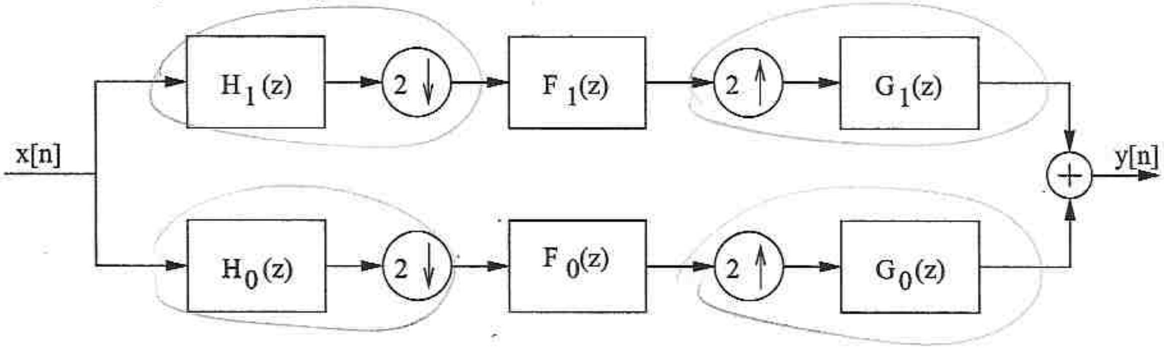


Figure 2: A two-channel filter bank.

For the filter bank in Figure 2, find the conditions for perfect reconstruction, i.e., the conditions on the filters $F_i(z), G_i(z), H_i(z)$ (for $i = 0, 1$) such that $y[n] = x[n]$.

$H_1(z) \sim$ gain 1, cutoff $\pi/2$ $\qquad$ $H_0(z) \sim$ gain 1, cutoff $\pi/2$

$G_1(z) \sim$ gain 2, cutoff $\pi/2$ $\qquad$ $G_0(z) \sim$ gain 2, cutoff $\pi/2$

0 / 15

$F_1(z), F_0(z)$ must ensure that the two branches add up to $y(n)$, i.e., each branch is $\frac{y(n)}{2}$ (by symmetry)

System

$$y(n) = \frac{x(n)}{2} \rightarrow H(z) = \frac{1}{2}$$

$F_1(z) \sim$ gain $1/2$, cutoff π $\qquad$ $F_0(z) \sim$ gain $1/2$, cutoff π

Keep all frequencies, scale the input uniformly

NOT SURE WHAT YOU ARE DOING...