Department of EECS - University of California at Berkeley
EECS126 - Probability and Random Processes - Fall 2003
Midterm No. 2: 11/14/2003

There are five questions, worth 20% each. Answer on these sheets. Show your work. Good luck.

Question 1. Let {X, Y, Z} be independent N(0, 1) random variables.
a. (14%) Calculate

$$E[3X + 5Y \mid 2X - Y, X + Z].$$

b. (6%) How does the expression change if X, Y, Z are i.i.d. N(1, 1)?

Question 2. 25%. Let X, Y be independent random variables uniformly distributed in [0, 1]. Calculate $L[Y^2 \mid 2X + Y]$.

Question 3. 15%. Let $\{X_n, n \geq 1\}$ be independent N(0, 1) random variables. Define $Y_{n+1} = aY_n + (1 - a)X_{n+1}$ for $n \geq 0$ where Y_0 is a $N(0, \sigma^2)$ random variable independent of $\{X_n, n \geq 0\}$. Calculate

$$E[Y_{n+m} | Y_0, Y_1, \ldots, Y_n]$$

for $m, n \geq 0$.

Hint: First argue that observing $\{Y_0, Y_1, \ldots, Y_n\}$ is the same as observing $\{Y_0, X_1, \ldots, X_n\}$.
Second, get an expression for Y_{n+m} in terms of $Y_0, X_1, \ldots, X_{n+m}$. Finally, use the independence of the basic random variables.

Question 4. 20%. Given θ, the random variables $\{X_n, n \geq 1\}$ are i.i.d. $U[0, \theta]$. Assume that θ is exponentially distributed with rate λ.

a. Find the MAP $\hat{\theta}_n$ of θ given $\{X_1, \ldots, X_n\}$.

b. Calculate $E(|\theta - \hat{\theta}_n|)$.

Question 5. 20%. Let (X, Y) be jointly Gaussian. Show that X – E[X | Y] is Gaussian and calculate its mean and variance.