Name: ________________________ Student ID No: ______________

UNIVERSITY OF CALIFORNIA

College of Engineering
Department of Electrical Engineering and
Computer Sciences

Professor Tse Fall 1998

EECS 126 — MIDTERM #1

2 October 1998, 11:10–12:10

[20 pts.] 1a. Suppose that E, F are events and $P(E) = 0.4$. What can you say about $P(E|F)$ if:

- **i)** E and F are independent?
- **ii)** E and F are mutually exclusive?
- **iii)** $F \subset E$?
- **iv)** $E \subset F$?

[10 pts.] b. If the occurrence of event B makes A more likely (i.e., $P(A|B) > P(A)$), then does the occurrence of event A make B more likely? Justify you answer.

[30 pts.] 2. There are 2 machines having lifetimes distributed with cdf's F_1 and F_2. Suppose one of the 2 machines is randomly picked with equal probability and put in operation at time 0. Conditional on the fact that the machine is still running at time t, what is the probability that it is machine 1 that was picked?

[20 pts.] 3a. Consider a binary channel with cross-over probability

$$P(\text{output} = 1|\text{input} = 0) = \varepsilon_1$$
$$P(\text{output} = 0|\text{input} = 1) = \varepsilon_2$$

Suppose $P(\text{input} = 0) = p$.

$$P(\text{input} = 1) = 1 - p$$

Further suppose you use a detection rule which decides that 0 is transmitted if 0 is received, and 1 is transmitted if 1 is received. Find the probability that you will make an error.

[20 pts.] b. Suppose now that $p = \frac{1}{2},\ \varepsilon_1 = \varepsilon_2 = \varepsilon < \frac{1}{2}$. A student thinks that a *random* detection rule can perform better than the detection rule above. Namely, the student flips a biased coin with $P(\text{head}) = \varepsilon$. If the coin lands on a tail, the student decides that what is transmitted is the *same* as what is received; if the coin lands on a head, he decides that what is transmitted is *opposite* to what is received. What is the probability that the student makes an error using this rule? Is this a better rule than the one in (a)?

Name: ___________________________ Student ID No: _______________