UNIVERSITY OF CALIFORNIA, BERKELEY
College of Engineering
Department of Electrical Engineering and Computer Sciences

EE 130: IC Devices — Spring 2007

FINAL EXAMINATION

NAME: SOLUTIONS ______________ ______________
(print) Last First Signature

STUDENT ID#: ______________

INSTRUCTIONS:

1. **Use the values of physical constants provided below.**
2. **SHOW YOUR WORK. (Make your methods clear to the grader!)**
3. **Clearly mark (underline or box) numeric answers. Specify the units on answers whenever appropriate.**

Physical Constants

Description	Symbol	Value
electronic charge	q	1.6×10^{-19} C
electron rest mass	m_o	9.1×10^{-31} kg
thermal voltage at 300K	kT/q	0.026 V
Boltzmann's constant	k	8.62×10^{-5} eV/K

$(kT/q)\ln(10) = 0.060$ V at $T = 300$K

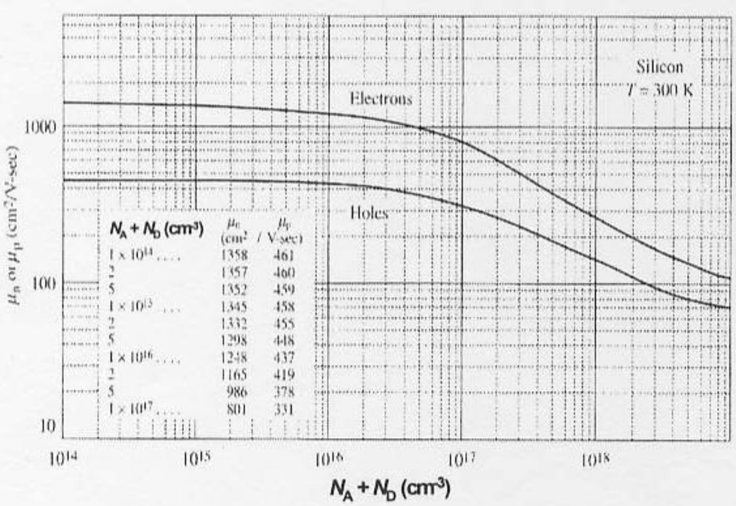
Properties of SiO_2 at 300K

Description	Symbol	Value
band gap	E_G	9 eV
permittivity	ε_{SiO2}	3.45×10^{-13} F/cm
electron affinity	χ_{SiO2}	0.95 V

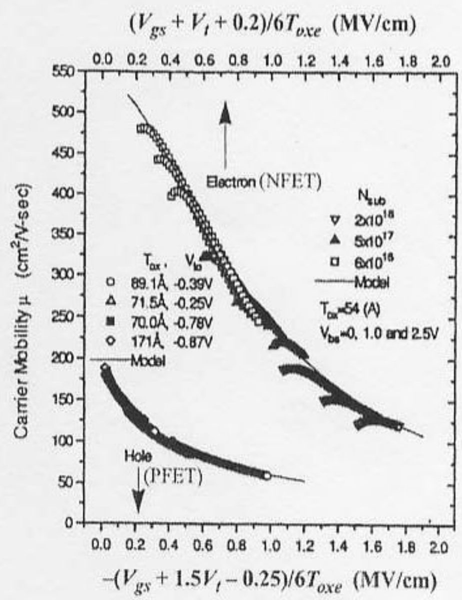
Properties of Silicon at 300K

Description	Symbol	Value
band gap	E_G	1.12 eV
intrinsic carrier density	n_i	10^{10} cm^{-3}
permittivity	ε_{Si}	1.0×10^{-12} F/cm
electron affinity	χ_{Si}	4.03 V

Electron and Hole Mobilities in Silicon at 300K



Field-Effect Mobilities in Si at 300K



SCORE:

1 ________ / 30

2 ________ / 30

3 ________ / 30

4 ________ / 30

5 ________ / 30

6 ________ / 30

Total: ________ / 180

Problem 1: Semiconductor Fundamentals [30 points]
Consider an uncompensated Si sample of length 0.5 mm, uniformly doped with boron ($N_A = 10^{17}$ cm^{-3}), maintained under equilibrium conditions at $T = 300$K.

a) What are the electron and hole concentrations, *n* and *p*? **[4 pts]** Boron is an acceptor in Si.

Since $N_A > N_D$, this sample is p-type.

$$p \simeq N_A = \boxed{10^{17}\ \text{cm}^{-3}}$$

$$n = \frac{n_i^2}{p} = \frac{(10^{10})^2}{10^{17}} = \boxed{10^3\ \text{cm}^{-3}}$$

b) Estimate the resistivity of this sample. **[5 pts]**

$$\rho = \frac{1}{q\mu_n n + q\mu_p p} \simeq \frac{1}{q\mu_p p} \quad \text{since } p \gg n$$

From the plot on Page 1, $\mu_p = 331\ \text{cm}^2/\text{V}\cdot\text{s}$

$$\rho = \left[1.6\times10^{-19}(331)(10^{17})\right]^{-1} \simeq \frac{1}{5} = \boxed{0.2\ \Omega\text{-cm}}$$

c) Given that the electron conductivity effective mass m_n^* is ~2×10^{-31} kg, estimate the average time between electron scattering events in this sample. **Note that 1 kg cm^2/V/s/C = 10^{-4} s. [5 pts]**

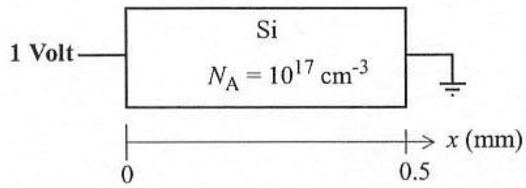
$$\mu_n = \frac{q\tau_{mn}}{m_n^*} \quad \Rightarrow \quad \tau_{mn} = \frac{\mu_n m_n^*}{q}$$

From the plot on Page 1, $\mu_n \simeq 800\ \text{cm}^2/\text{V}\cdot\text{s}$

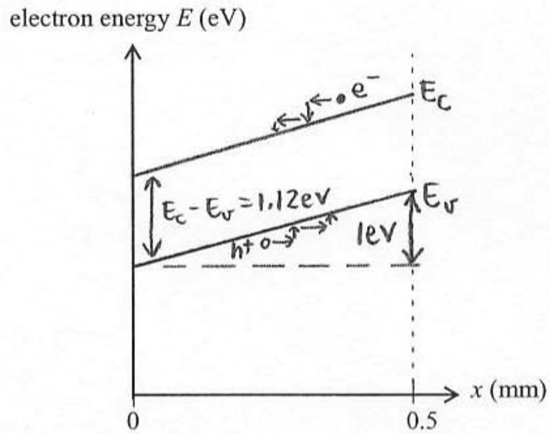
$$\tau_{mn} = \frac{800\cdot 2\times10^{-31}}{1.6\times10^{-19}} = \frac{1.6\times10^{-28}}{1.6\times10^{-19}} = 10^{-9}(10^{-4}\text{s}) = 10^{-3}\text{s}$$

$$\boxed{\tau_{mn} = 0.1\ \text{ps}}$$

d) Suppose a potential difference of 1 V is applied across the sample as shown below:



i) Sketch the non-equilibrium energy-band diagram on the plot below, showing E_c & E_v as a function of distance x. Indicate values for E_c-E_v and the total amount of energy-band bending across the sample. **Illustrate electron drift and hole drift on your diagram. [7 pts]**



ii) What are the hole and electron drift velocities? **[6 pts]**

$v_d = \mu \mathcal{E}$ $\qquad$ $\mathcal{E} = \dfrac{1V}{0.5\times10^{-1}\text{cm}} = 20\ \text{V/cm}$

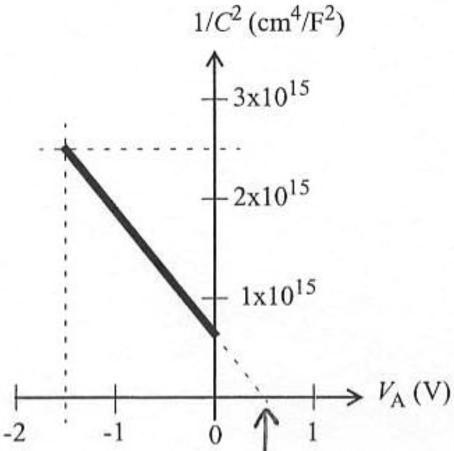
hole drift velocity $v_{d,p} = (331\ \text{cm}^2/\text{V}\cdot\text{s})(20\ \text{V/cm}) = \boxed{6620\ \text{cm/s}}$

electron drift velocity $v_{d,n} = 800 \cdot 20 = \boxed{16{,}000\ \text{cm/s}}$

iii) How would the drift velocities change if the temperature were to be increased (*e.g.* to 100°C)? Explain briefly. **[3 pts]**

v_d would decrease because μ would decrease due to increased phonon scattering.

Problem 2: Metal-Semiconductor Contact [30 points]
A Schottky diode formed on n-type Si at $T = 300\text{K}$ yields the $1/C^2$ vs. V_A plot shown below. (C is the small-signal capacitance per cm^2.)



0.5V

a) What is the built-in potential V_{bi}? **[3 pts]**

$$\frac{1}{C^2} = \frac{2}{qN_D\varepsilon_{Si}}(V_{bi} - V_A) \rightarrow 0 \text{ at } V_A = V_{bi} \quad \Rightarrow \boxed{V_{bi} = 0.5\text{V}}$$

b) What is the doping concentration N_D in the Si? **[6 pts]**

$$\text{slope of curve} = \frac{-2}{qN_D\varepsilon_{Si}} = \frac{-2.5\times10^{15}}{2} = -1.25\times10^{15}\ \text{cm}^4/\text{F}^2/\text{V}$$

$$N_D = \frac{2}{1.6\times10^{-19}(10^{-12})(1.25\times10^{15})} \equiv \frac{2}{2\times10^{-16}} = \boxed{10^{16}\ \text{cm}^{-3}}$$

c) What is the Schottky barrier height Φ_{Bn}? **[6 pts]**

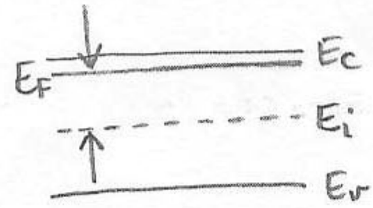
$$qV_{bi} = \Phi_{Bn} - (E_c - E_F)_{FB} = 0.5\text{eV}$$

$$\Phi_{Bn} = qV_{bi} + (E_c - E_F)_{FB}$$

$$= 0.5\text{eV} + 0.2\text{eV}$$

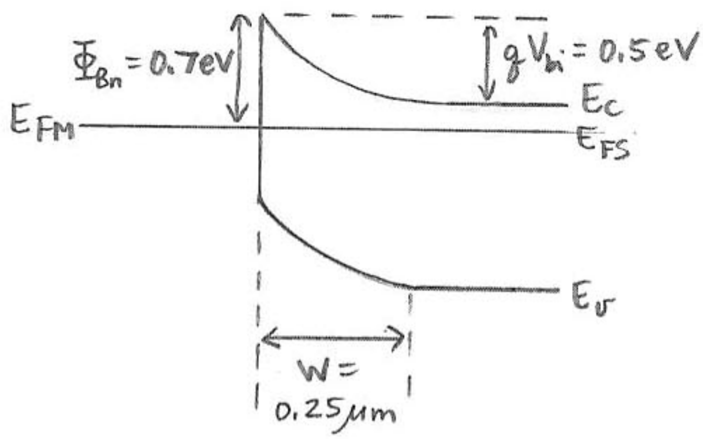
$$= \boxed{0.7\ \text{eV}}$$

$$E_F - E_i = kT\ln\left(\frac{n}{n_i}\right) = kT\ln\left(\frac{10^{16}}{10^{10}}\right)$$
$$= 6\,kT\ln(10)$$
$$= 6\times0.06\text{eV}$$
$$= \underline{0.36\text{eV}}$$



$$(E_c - E_F)_{FB} = \frac{E_G}{2} - (E_F - E_i)$$
$$= 0.56 - 0.36 = \underline{0.2\text{eV}}$$

d) Draw the equilibrium energy-band diagram for the Schottky diode, showing E_c, E_v, and E_F in the Si. Label Φ_{Bn}, qV_{bi}, and the depletion width W. (Numerical values are required.) **[10 pts]**



$$W = \sqrt{\frac{2\varepsilon_{Si} V_{bi}}{q N_D}} = \sqrt{\frac{2(10^{-12})(0.5)}{1.6\times10^{-19}(10^{16})}} = \sqrt{\frac{10^{-8}}{16}} = \frac{1}{4}\times 10^{-4} = \boxed{0.25\times10^{-4}\,\text{cm}}$$

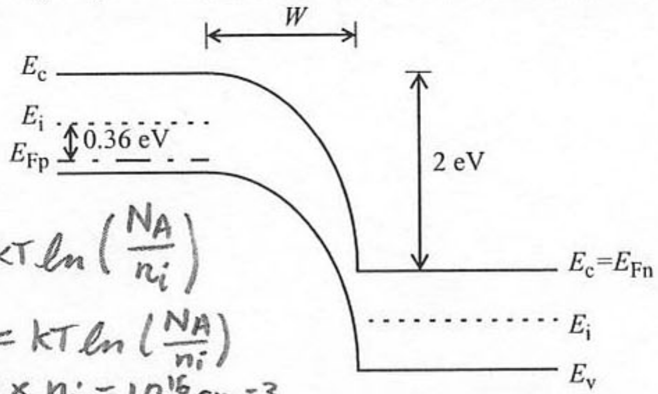
e) Describe two approaches to achieve low specific contact resistivity (ρ_c) in a practical ohmic contact. **[5 pts]**

To achieve a practical ohmic contact, carriers must easily tunnel through the Schottky barrier. To allow for this, the Schottky barrier height should be lowered as much as possible (e.g. by appropriate choice of metal material) and the depletion width should be very small (less than 10 nm), which is achieved by heavily doping the semiconductor near/at the contact.

Problem 3 [30 points]

The energy-band diagram for a Si p-n$^+$ junction diode maintained at 300K is shown below.

$kT\,ln(10) = 0.06$ eV



$$(E_i - E_F)_{p\text{-side}} = kT \ln\left(\frac{N_A}{n_i}\right)$$

$$0.36\,\text{eV} = 6 \times kT \ln(10) = kT \ln\left(\frac{N_A}{n_i}\right)$$

$$\Rightarrow N_A = 10^6 \times n_i = \underline{10^{16}\,\text{cm}^{-3}}$$

a) What is the built-in potential, V_{bi}? **[4 pts]**

$$qV_{bi} = (E_i - E_F)_{p\text{-side}} + (E_F - E_i)_{n\text{-side}}$$

$$= 0.36\text{ eV} + \frac{1.12\text{eV}}{2} = 0.92\text{eV}$$

$$\boxed{V_{bi} = 0.92\text{V}}$$

b) What is the applied bias, V_A? **[3 pts]**

Total band bending $= q(V_{bi} - V_A) = 2\text{eV}$

$$\Rightarrow V_{bi} - V_A = 2\text{V}$$

$$V_A = V_{bi} - 2 = 0.92 - 2 = \boxed{-1.08\text{V}}$$

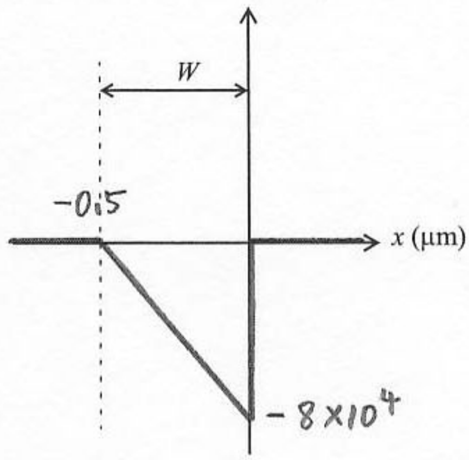
c) What is the small-signal junction capacitance (in units of Farads/cm^2) at this bias? **[6 pts]**

$$C_J = \frac{\varepsilon_{Si}}{W} = \frac{10^{-12}}{0.5\times10^{-4}} = \boxed{2\times10^{-8}\text{ F/cm}^2}$$

$$W = \sqrt{\frac{2\varepsilon_s(V_{bi}-V_A)}{qN_D}} = \sqrt{\frac{2(10^{-12})(2)}{1.6\times10^{-19}(10^{16})}} = \sqrt{\frac{10^{-8}}{4}} = \frac{1}{2}\times10^{-4}$$

$$= \underline{\underline{0.5\times10^{-4}\text{ cm}}}$$

d) Sketch the electric field distribution in the diode, indicating the numerical value of the peak electric field. **[8 pts]**



$$\frac{1}{2} \mathcal{E}_{peak} W = V_{bi} - V_A$$

$$\mathcal{E}_{peak} = \frac{2(V_{bi} - V_A)}{W}$$

$$= \frac{2(2)}{0.5 \times 10^{-4}} = 8 \times 10^4 \text{ V/cm}$$

e) How does generation-recombination in the depletion region affect the diode current under this bias condition? Explain briefly. **[5 pts]**

Since the diode is reverse biased, $pn < n_i^2$ in the depletion region.

Thus, there is net generation in the depletion region.

Generated holes are swept into the p-side
Generated electrons are swept into the n-side
$\Rightarrow$ additional negative diode current

$\therefore$ R-G increases the magnitude of the diode current.

f) Suppose that the criticial electric field for breakdown is $\mathcal{E}_{CR} = 5 \times 10^5$ V/cm. What is the dominant mechanism by which breakdown will occur, as the reverse bias is increased? Explain briefly. **[4 pts]**

$|\mathcal{E}_{peak}| < \mathcal{E}_{crit}$

$\therefore |V_{BR}| > V_A$ so $W > 0.5\ \mu m$ at breakdown.

Carriers cannot easily tunnel through such a wide depletion region, for the zener breakdown mechanism to occur.

$\therefore$ Breakdown will occur by the avalanche (impact ionization) mechanism.

Problem 4: Bipolar Junction Transistor [30 points]

a) Short-Answer Questions

i) The base width W_B is a critical parameter which affects BJT performance. Describe the W_B design tradeoff, by explaining why W_B should not be too small (give at least 1 reason) or too large (give at least 1 reason). **[4 pts]**

W_B should not be too small to ensure
- large Early voltage (small base-width modulation effect)
- large punchthrough voltage

W_B should not be too large to ensure
- high dc current gain (base transport factor ~ 1)
- high f_T (short base transit time)

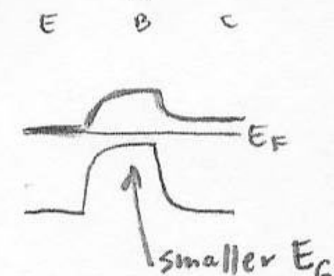
ii) Describe qualitatively (without the use of any formulas or equations -- but perhaps with the aid of an energy band diagram) why the use of silicon-germanium (rather than Si) as a base material enhances the DC current gain of a Si npn BJT. **[4 pts]**

The energy bandgap is smaller for $Si_{1-x}Ge_x$.

→ Barrier height for hole injection into the emitter is larger than the barrier height for electron injection into the base.

→ Emitter injection efficiency is improved

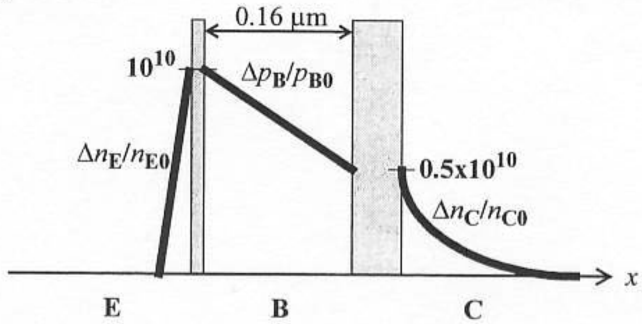
→ β_{dc} improves



iii) How can the minority carrier lifetime in the base be reduced? Why would this be desirable, for certain BJT applications? **[4 pts]**

τ_B is reduced by increasing the density of trap states, N_T. This is achieved in practice by incorporating impurities such as gold atoms in the base, which have associated allowed energy level near the midgap of Si. This is desirable if we want to improve the turn-off time for the BJT, i.e. improve switching speed.

b) The normalized excess minority carrier concentrations in the quasi-neutral regions of a pnp BJT maintained at T = 300K are shown in the figure below.



i) What is the biasing mode of this transistor? Justify your answer. **[3 pts]**

Since the excess carrier concentrations at the edges of the depletion region for both the emitter junction and the collector junction are greater than zero, both junctions are forward biased. ⇒ This BJT is operating in saturation mode.

ii) Estimate the value of V_{EB}. **[3 pts]**
(Recall that $(kT/q)(\ln 10) = 60$ mV)

$$\frac{\Delta p_B}{p_{B0}} = e^{qV_{EB}/kT} - 1 = 10^{10} \Rightarrow e^{qV_{EB}/kT} \approx 10^{10}$$

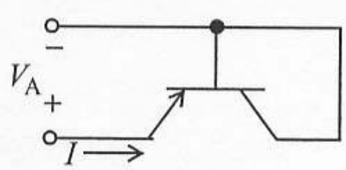
$$V_{EB} = \frac{kT}{q}\ln(10^{10}) = 10\times\frac{kT}{q}\ln(10) = \boxed{0.6\text{V}}$$

iii) Estimate the collector current density (J_C) for this BJT when it is biased at the edge of saturation (V_{CB} = 0V, V_{EB} as shown above), if $N_B = 10^{18}$ cm^{-3} and D_B = 4 cm^2/s. **[6 pts]**

$$J_c = q\left(\frac{n_i^2}{N_B}\right)\frac{D_B}{W}\left(e^{qV_{EB}/kT}-1\right) = 1.6\times10^{-19}\left(\frac{10^{20}}{10^{18}}\right)\left(\frac{4}{0.16\times10^{-4}}\right)(10^{10})$$

$$= 40\times10^{-3} = \boxed{40 \text{ mA/cm}^2}$$

c) Derive a simple I-V equation for an pnp BJT connected as a diode as shown below. I should be expressed only in terms of V_A and the Ebers-Moll parameters (α_F, α_R, I_{F0}, I_{R0}). **[6 pts]**



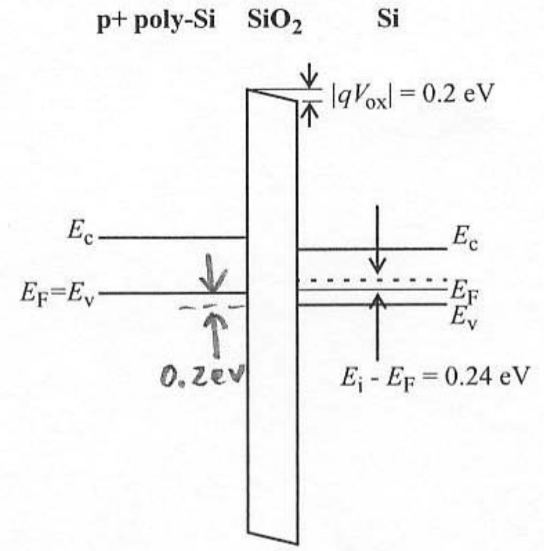
$$I = I_E = I_{F0}\left(e^{qV_{EB}/kT}-1\right) - \alpha_R I_{R0}\left(e^{qV_{CB}/kT}-1\right)$$

$$V_A = V_{EB}$$

$$V_{CB} = 0 \Rightarrow \left(e^{qV_{CB}/kT}-1\right) = 0$$

$$\therefore \boxed{I = I_{F0}\left(e^{qV_A/kT}-1\right)}$$

Problem 5: Metal-Oxide-Semiconductor Capacitor [30 points]
The flat-band energy-band diagram for a p+ poly-Si gated capacitor of area 10^{-4} cm^2 and $T_{oxe} = 3.45$ nm, maintained at $T = 300$K, is shown below:



$$E_F - E_v = \frac{E_G}{2} - 0.24\text{eV} = \frac{1.12\text{eV}}{2} - 0.24\text{eV} = \underline{\underline{0.32\text{ eV}}}$$

a) What is the flatband voltage V_{FB}? **[5 pts]**

qV_{FB} is the difference between the Fermi level in the gate and the Fermi level in the semiconductor:

$$qV_{FB} = \Phi_M - \Phi_S - \frac{Q_F}{C_{ox}}$$

$$= 5.15 - 4.83 - 0.2 = 0.12\text{eV}$$

$$\boxed{V_{FB} = 0.12\text{V}}$$

$$\Phi_M = \chi_{Si} + E_{G,Si} = 4.03 + 1.12 = 5.15\text{eV}$$

$$\Phi_S = \chi_{Si} + \frac{E_{G,Si}}{2} + (E_i - E_F)_{Si} = 4.03 + 0.56 + 0.24 = 4.83\text{ eV}$$

b) Calculate the oxide fixed charge density Q_F (in units of C/cm^2). **[5 pts]**

The voltage drop across the oxide at flatband is due to the oxide charge.

$$\frac{Q_f}{C_{ox}} = 0.2\text{V}$$

$$C_{ox} = \frac{\varepsilon_{ox}}{T_{ox}} = \frac{3.45\times10^{-13}\text{ F/cm}}{3.45\times10^{-7}\text{ cm}} = 10^{-6}\text{ F/cm}^2$$

$$Q_f = 0.2\times10^{-6} = \underline{\underline{2\times10^{-7}\text{ C/cm}^2}}$$

c) Calculate (approximately) the threshold voltage, V_T. **[6 pts]**

$$V_T = V_{FB} + 2\phi_F + \frac{Q_{dep}}{C_{ox}}$$

$$= +0.12 + 2(0.24) + \underbrace{\frac{4\times10^{-9}}{10^{-6}}}_{\text{negligible}}$$

$$= \boxed{0.6\ \text{V}}$$

$$\phi_F = \frac{E_i - E_F}{q} = 0.24\text{V} = 4\times 0.06\text{V}$$

$$= \frac{kT}{q}\ln\left(\frac{N_A}{n_i}\right) = 4\times\frac{kT}{q}\ln(10)$$

$$\Rightarrow \ln\left(\frac{N_A}{n_i}\right) = \ln(10^4)$$

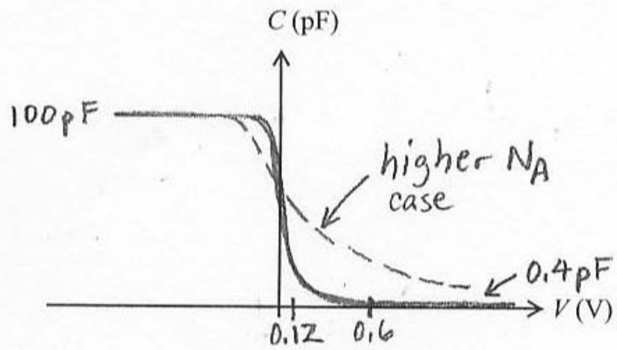
$$N_A = 10^4 n_i = 10^{14}\ \text{cm}^{-3}$$

$$Q_{dep} = \sqrt{2\varepsilon_{Si}\, q N_A (2\phi_F)}$$

$$= \sqrt{2(10^{-12})(1.6\times10^{-19})(10^{14})(0.48)}$$

$$\cong \sqrt{16\times10^{-18}} = 4\times10^{-9}\ \text{C/cm}^2$$

d) Draw the high-frequency *C-V* curve for this capacitor, indicating the maximum and minimum capacitance values on your plot, as well as V_{FB} and V_T (consistent with your answers to parts (a) and (c), respectively). **[9 pts]**



$$C_{max} = AC_{ox} = 10^{-4}\text{cm}^2\,(10^{-6}\,\text{F/cm}^2)$$

$$= 10^{-10}\text{F} = \boxed{100\text{pF}}$$

$$C_{dep} = \frac{\varepsilon_{Si}}{W_T} = \frac{10^{-12}\ \text{F/cm}}{0.25\times10^{-3}\text{cm}} = 4\times10^{-9}\ \text{F/cm}^2$$

$$W_T = \sqrt{\frac{2\varepsilon_{Si}(2\phi_F)}{qN}} = \sqrt{\frac{2(10^{-12})(0.48)}{1.6\times10^{-19}(10^{14})}} = \sqrt{\frac{10^{-6}}{16}} = \frac{1}{4}\times10^{-3} = 0.25\times10^{-3}\ \text{cm}$$

$$\frac{A}{C_{min}} = \frac{1}{C_{ox}} + \frac{1}{C_{dep}} = \frac{1}{10^{-6}} + \frac{1}{4\times10^{-9}} \simeq \frac{1}{4\times10^{-9}}$$

$$C_{min} = A\times 4\times10^{-9} = 10^{-4}\times4\times10^{-9} = \boxed{0.4\ \text{pF}}$$

e) Indicate **with a dashed line** on your plot in part (d) how the *C-V* curve would change if the dopant concentration in the Si were to be increased significantly. (Consider the qualitative impact on V_{FB}, V_T, and C_{min}.) **[5 pts]**

- V_{FB} would be more negative since $\Phi_S \uparrow$
- V_T would increase since V_{FB}, ϕ_F, Q_{dep} all increase
- C_{min} would increase because $W_T \downarrow$

Problem 6: MOS Field-Effect Transistor [30 points]

a) In a certain CMOS technology, the electrical oxide thickness is $T_{oxe} = 5$ nm (so that $C_{oxe} = 6.9\times10^{-7}$ F/cm^2), the bulk-charge factor $m = 1.5$, and the threshold voltage of a long-channel MOSFET is $|V_T| = 0.5$ V.

i) What is the average inversion-layer electron mobility in an n-channel MOSFET, for a gate bias $V_{GS} = 1.5$ V? **[4 pts]**

Effective vertical electric field $\mathcal{E}_{eff} = \dfrac{V_{GS} + V_T + 0.2}{6T_{oxe}}$

$$\mathcal{E}_{eff} = \frac{1.5+0.5+0.2}{6\cdot 5\times10^{-7}} = \frac{2.2\text{ V}}{3\times10^{-6}\text{cm}} \simeq 0.7\text{ MV/cm}$$

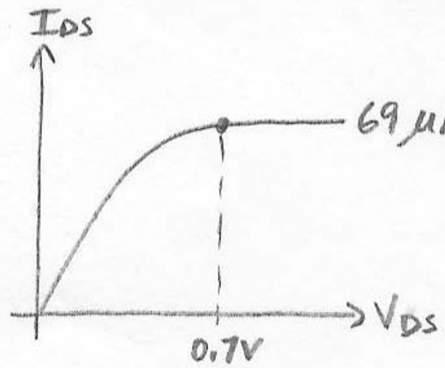
From plot on Page 1, $\underline{\mu_{n,eff} \approx 325\text{ cm}^2/\text{V}\cdot\text{s}} \simeq 300\text{ cm}^2/\text{V}\cdot\text{s}$

ii) Sketch the I_{DS} *vs.* V_{DS} characteristic for an n-channel MOSFET of channel width $W = 10$ μm, channel length $L = 10$ μm, and gate bias $V_{GS} = 1.5$ V. Indicate the values of V_{Dsat} and I_{Dsat}. **[6 pts]**

$$I_{Dsat} = \frac{\mu_{n,eff} C_{ox}}{2m}\frac{W}{L}(V_{GS}-V_T)^2$$

$$\cong \frac{300\cdot 6.9\times10^{-7}}{2\cdot 1.5}\left(\frac{10}{10}\right)(1.5-0.5)^2 = 6.9\times10^{-5}\text{ A} = \underline{69\,\mu\text{A}}$$

$$V_{Dsat} = \frac{V_G - V_T}{m} = \frac{1.5-1}{1.5} \cong 0.7\text{ V}$$



iii) For what channel lengths will the effect of velocity saturation be significant (*i.e.* resulting in a reduction in I_{Dsat} by more than a factor of 2)? $v_{sat} = 8\times10^6$ cm/s. **[5 pts]**

saturation E-field $\mathcal{E}_{sat} = \dfrac{2v_{sat}}{\mu_{n,eff}} = \dfrac{2\times8\times10^6}{300} \cong 5\times10^4\text{ V/cm}$

Velocity saturation will be significant when

$$\mathcal{E}_{sat}L \leq \frac{V_{GS}-V_T}{m} \cong 0.7\text{ V}$$

$$L \leq \frac{0.7\text{ V}}{5\times10^4\text{ V/cm}} = 1.4\times10^{-5}\text{ cm}$$

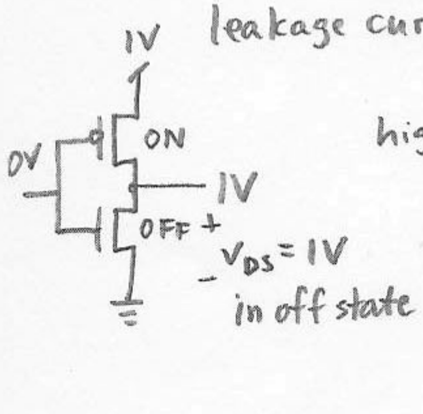
$$\Rightarrow \boxed{L \leq 0.14\,\mu\text{m}}$$

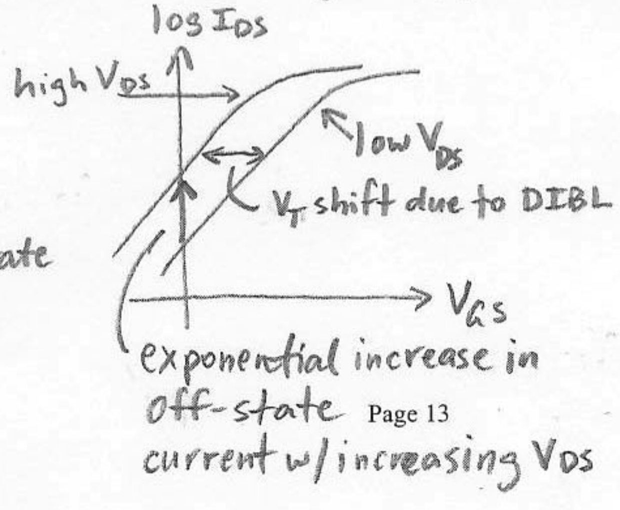
b) Indicate in the table below (by checking the appropriate box for each line) the effect of **decreasing the effective oxide thickness** (T_{oxe}) on the performance parameters of an n-channel MOSFET. Provide brief, qualitative justification for each of your answers. **[9 pts]**

MOSFET parameter	**increases**	**decreases**	**remains the same**	**Qualitative Explanation (no formulas/equations!)**
Transconductance (g_m)	✓			Channel potential is coupled more strongly to gate voltage, so I_{DS} is a more sensitive function of V_{GS} (and Q_{inv} is larger for a given $V_{GS}-V_T$) $\therefore g_m \equiv \frac{\partial I_{DS}}{\partial V_{GS}}$ increases
Body effect parameter (γ)		✓		Channel potential is coupled more strongly to the gate voltage vs. the body voltage, so the body effect is reduced. ↑ impact of body bias on channel potential
Subthreshold swing (S)		✓		Gate voltage control over the channel potential is improved because of improved capacitive coupling. vs. drain voltage and body voltage

c) Why is drain-induced barrier lowering undesirable? (Hint: Consider how a MOSFET is typically biased in the off state, in CMOS circuits.) Describe 2 ways to reduce it. **[6 pts]**

DIBL is undesirable because it increases off-state ($V_{GS}=0V$) leakage current at large V_{DS}, relevant for CMOS applications.





- To reduce DIBL, reduce E-field penetration into the channel:
  - increase channel/body doping
  - decrease S/D junction depth (r_j)
  - reduce drain doping (use LDD structure)
  - reduce T_{oxe}