Midterm #1 Solutions – EECS 145L Fall 2005

1a

$V_0 = a V_+ + bV_-$
$V_0 = G_\pm (V_+ - V_-) + G_C (V_+ + V_-)/2$
$a V_+ + bV_- = (G_\pm + G_C/2) V_+ + (-G_\pm + G_C/2) V_-$
$a = G_\pm + G_C/2$
$b = -G_\pm + G_C/2$
Adding, $G_C = a + b$
Subtracting, $G_\pm = (a - b)/2$

Alternative solution:
Since the common mode gain is the change in V_0 per unit change in $(V_+ + V_-)/2$, we can add 1 V to both V_+ and V_- and see that $\Delta V_0 = a + b$. So $G_C = a + b$
Since the differential gain is the change in V_0 per unit change in $(V_+ - V_-)$, we can add 0.5 V to V_+, subtract 0.5 V from V_- and see that $\Delta V_0 = a/2 - b/2$. So $G_\pm = (a - b)/2$

2a

	Op Amp	Inverting op-amp circuit amplifier	Non-inverting op-amp circuit amplifier	Differential op-amp circuit amplifier	Instrumentation amplifier
High Z_{in}	YES	NO	YES	NO	YES
Differential input	YES	NO	NO	YES	YES
Defined gain over a frequency band	NO	YES	YES	YES	YES

[1 point off for each wrong answer]

3a

At 10 Hz, $A = 10^5$ and the op-amp equation gives $V_3 = -V_0/10^5$ (virtual ground)
$(V_1 - V_2)/100\ k\Omega - V_2/1\ k\Omega - V_2/1\ k\Omega = 0$
$V_1 - V_2 - 200V_2 = 0$
$\boxed{V_2 = V_1/201}$
$V_2/1\ k\Omega + V_0 / 100\ k\Omega = 0$
$100\ V_2 + V_0 = 0$
$\boxed{V_0 = -100\ V_2 \approx -0.5\ V_1}$
$\boxed{V_3 = -0.5 \times 10^{-5}\ V_1 \quad (\approx 0 \text{ was also accepted})}$

3b

At 1 MHz, A = 1, the op-amp equation gives $V_0 = -V_3$
$(V_1 - V_2)/100\ k\Omega + (V_3 - V_2)/1\ k\Omega - V_2/1\ k\Omega = 0$
$V_1 - V_2 + 100V_3 - 100V_2 - 100\ V_2 = 0$
$V_1 + 100V_3 - 201V_2 = 0$

$(V_2 - V_3)/1\ \text{k}\Omega + (V_0 - V_3)/100\ \text{k}\Omega = 0$

$100V_2 - 100V_3 + V_0 - V_3 = 0$

$100V_2 - 102V_3 = 0$

$V_2 \approx V_3$

$V_1 \approx 100V_3 \approx 100V_2 \approx -100\ V_0$

$\boxed{V_3 \approx V_1/100}$

$\boxed{V_2 \approx V_1/100}$

$\boxed{V_0 \approx -V_1/100}$

Alternative solution: solve for any value of A and plug in for 10 Hz and 10^6 Hz

Op-amp equation $V_0 = -AV_3$

Kirchhoff's current law at node V_2: $\dfrac{V_1 - V_2}{100\ \text{k}\Omega} + \dfrac{V_3 - V_2}{1\ \text{k}\Omega} + \dfrac{0 - V_2}{1\ \text{k}\Omega} = 0$

$V_1 = V_2 + 100V_2 - 100V_3 + 100V_2 = 201V_2 - 100V_3$

Kirchhoff's current law at node V_3: $\dfrac{V_2 - V_3}{1\ \text{k}\Omega} + \dfrac{V_0 - V_3}{100\ \text{k}\Omega} = 0$

$100V_2 = 100V_3 + V_3 - V_0 = (101 + A)V_3$

$$V_1 = \left[\frac{201(101 + A)}{100} - 100\right]V_3 = \left[\frac{10301 + 201A}{100}\right]V_3$$

$$V_1 = \left[201 - \frac{100(100)}{101 + A}\right]V_2 = \left[\frac{10301 + 201A}{101 + A}\right]V_2$$

$$\boxed{V_2 = \frac{(101 + A)V_1}{10301 + 201A} \approx \frac{1 + A/100}{100 + 2A}V_1} \quad \boxed{V_3 = \frac{100V_1}{10301 + 210A} \approx \frac{V_1}{100 + 2A}}$$

$$\boxed{V_0 = \frac{-100AV_1}{10301 + 201A} \approx \frac{-AV_1}{100 + 2A}}$$

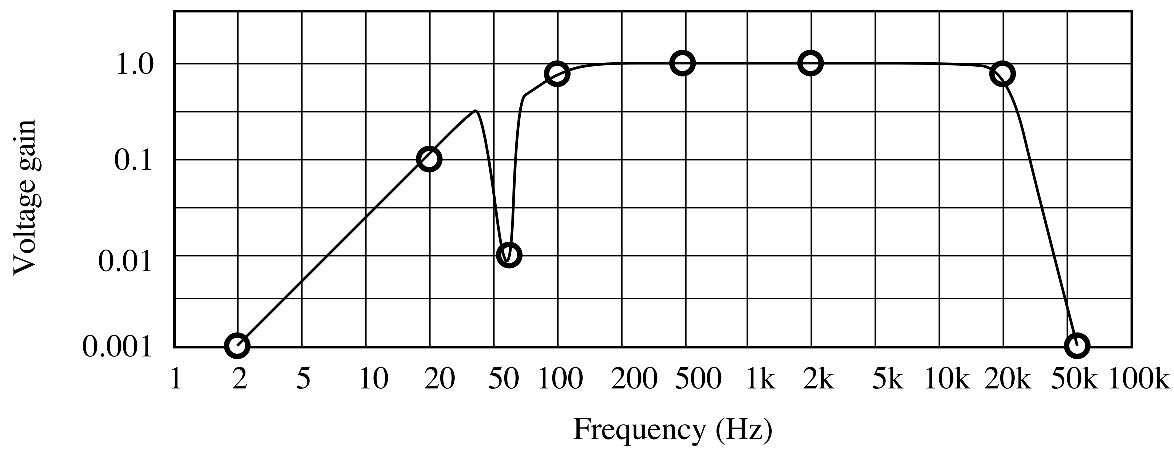
f = 10 Hz, $A = 10^5$

$V_2 \approx V_1/201 \approx 5 \times 10^{-3}\ V_1$ $\quad V_3 \approx 100V_1/(201 \times 10^5) \approx 5 \times 10^{-6}\ V_1$ $\quad V_0 \approx -0.5\ V_1$

f = 1 MHz, $A = 1$

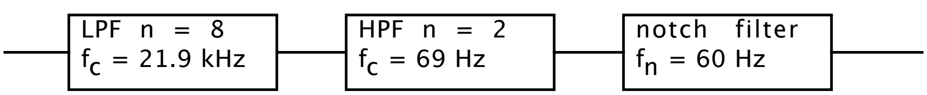
$V_2 \approx 100V_1/10000 \approx 10^{-2}\ V_1$ $\quad V_3 \approx 100V_1/10000 \approx 10^{-2}\ V_1$ $\quad V_0 \approx -10^{-2}\ V_1$

4a



[1 point off for showing a constant gain of 0.001 below 2 Hz and above 55 kHz]

4b



The LPF needs to have a gain $G_1 = 0.9$ at $f_1 = 20$ kHz and drop to a gain $G_2 < 0.001$ at $f_2 = 55$ kHz. Assuming that the corner frequency is near 20 kHz, find the smallest value of n for which the gain =0.001 occurs at a value of f2/fc less than 55 kHz/20 kHz = 2.75. Looking at the LPF table, we see that f/fc = 3.162 at n = 6, f/fc = 2.371 at n = 8, and f/fc = 1.995 at n = 10.

Alternatively, we can use the fact that for a LPF with $G_2 << 1$, $G_2 \approx (f/fc)^{-n}$ and $n = -\ln(G_2)/\ln(f/f_c) = -\ln(0.001)/\ln(55/20) = 6.83$.

Choosing n = 8, G1 = 0.9 is at f1/fc = 20 kHz/fc = 0.913, and fc = 21,906 Hz
G2 = 0.001 is at f2/fc = 2.371 from which we compute f2 = 51,939 Hz, which is less than 55 kHz as required.

LPF n = 8, f_c = 21.91 kHz

The HPF needs to have a gain $G_1 = 0.9$ at 100 Hz and drop to a gain $G_2 = 0.001$ at 2 Hz.

Assuming that the corner frequency is near 100 Hz, find the smallest value of n for which the gain =0.001 occurs at a value of f2/fc greater than 2 Hz/100 Hz = 0.02. Looking at the HPF table, we see that f/fc = 0.032 at n = 2 and f/fc = 0.178 at n = 4.

Alternatively, we can use the fact that for a HPF with $G_2 << 1$, $G_2 \approx (f/fc)^n$ and $n = \ln(G_2)/\ln(f/f_c) = \ln(0.001)/\ln(2/100) = 1.77$.

Choosing n = 2, G_1 = 0.9 is at f_1/f_c = 100 Hz/f_c = 1.437, and f_c = 69 Hz
G2 = 0.001 is at f2/fc = 0.032 from which we compute f2 = 2.2 Hz, which is greater than 2 Hz as required.

HPF n = 2, f_c = 69 Hz

n = 4, fc = 83 Hz was also accepted

The HPF has a gain just a bit below 0.7 at 60 Hz and does not meet the gain requirement of 0.01. A notch filter with accurate components should provide the necessary low gain.

[3 points off for using a 10 or 12 pole HPF rather than a notch filter to reduce the gain from 0.9 at 100 Hz to 0.01 at 60 Hz- this uses 4 or 5 more op-amps, is inefficient, and has more components that can fail]

145L midterm #1 grade distribution:

Problem	
1	13.0 (3.5 rms) (15 max)
2	14.4 (1.2 rms) (15 max)
3	32.3 (4.6 rms) (35 max)
4	31.7 (5.0 rms) (35 max)

maximum score = 100
average score = 91.4 (9.0 rms)

65-69	0	F
70-74	1	D
75-79	2	C
80-84	1	C
85-89	2	B
90-94	3	B
95-99	5	A
100	5	A

GPA 3.26