EECS 20N: Structure and Interpretation of Signals and Systems — MIDTERM 2

Department of Electrical Engineering and Computer Sciences — 24 October 2006

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EECS 20N, Fall, 2006, Midterm 2, Babak

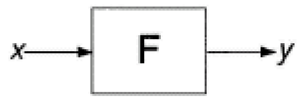
Basic Formulas:

Discrete Fourier Series (DFS) Complex exponential Fourier series synthesis and analysis equations for a periodic discrete-time signal having period *p*:

$$x(n) = \sum_{k=\langle p \rangle} X_k e^{ik\omega_0 n} \qquad \longleftrightarrow \qquad X_k = \frac{1}{p} \sum_{n=\langle p \rangle} x(n) e^{-ik\omega_0 n},$$

where $p = \dfrac{2\pi}{\omega_0}$ and $\langle p \rangle$ denotes a suitable discrete interval of length *p* (i.e., an interval containing *p* continuous integers). For example, $\sum_{k=\langle p \rangle}$ may denote $\sum_{k=0}^{p-1}$ or $\sum_{k=1}^{p}$.

MT2.1(20 Points) Consider a continuous-time system $F : [\mathbb{R} \to \mathbb{C}] \to [\mathbb{R} \to \mathbb{C}]$ having input signal x and output signal y, as shown below:



This system takes the real part of its input signal:

$$y = F(x) = \text{Re}(x).$$

In other words,

$$\forall t \in \mathbb{R}, \qquad y(t) = \text{Re}(x)$$

Where $\text{Re}(\bullet)$ denotes taking the real part of a number. For each part below, you must explain your reasoning succinctly, but clearly and convincingly.

(a) Select the strongest true assertion from the list below.

(i) The system must be memoryless.

(ii) The system could be memoryless, but does not have to be.

(iii) The system cannot be memoryless.

(b) Select the strongest true assertion from the list below.

(i) The system must be causal.

(ii) The system could be causal, but does not have to be.

(iii) The system cannot be causal.

(c) Select the strongest true assertion from the list below.

(i) The system must be time invariant.

(ii) The system could be time invariant, but does not have to be.

(iii) The system cannot be time invariant.

(d) Select the strongest true assertion from the list below.

(i) The system must be linear.

(ii) The system could be linear, but does not have to be.

(iii) The system cannot be linear.

MT2.2 (25 Points) The unit-step response[1] s of a discrete-time linear, time-invariant system is given by:

$$\forall n \in \mathbb{Z}, \quad s(n) = (n+1)u(n),$$

Where u is the unit-step signal characterized as follows:

$$\forall n \in \mathbb{Z}, \quad u(n) = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0. \end{cases}$$

Explain your reasoning for each part succinctly, but clearly and convincingly.

(a) Determine and provide a well-labeled sketch of h, the impulse response of the system.

[1] Recall that the unit-step response of a system is, as the name suggests, the response of the system to the unit-step input signal.

(b) Select the strongest true assertion from the list below.

(i) The system must be memoryless.

(ii) The system could be memoryless, but does not have to be.

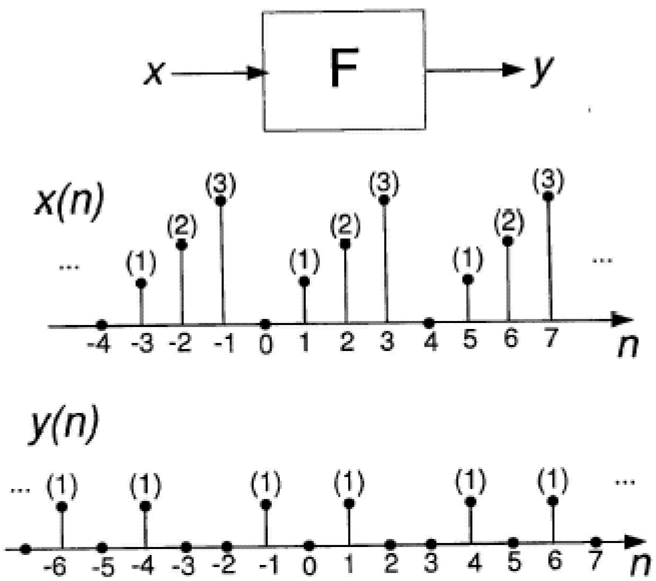
(iii) The system cannot be memoryless.

(c) Determine a simple expression for

$$\sum_{m=-\infty}^{n} h(m) .$$

Hint: Your answer will depend on n. You should be able to solve this part even without knowing the impulse response h from part (a).

MT2.3 (25 Points) Consider a discrete-time system $\mathsf{F} : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ having a periodic input signal x and a corresponding periodic output signal y, as shown below:



(a) Determine (p_x, ω_x) and (p_y, ω_y), the period and fundamental frequency of x and y, respectively.

(b) Determine the complex exponential discrete Fourier series (DFS) representation of the output signal y. In particular, determine a simple expression for the coefficients Y_k in the DFS expansion

$$Y_k = \frac{1}{p_y} \sum_{n=\langle p_y \rangle} y(n) e^{-ik\omega_y n}.$$

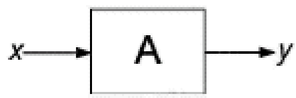
(c) Select the strongest true assertion from the list below. Explain your reasoning succinctly, but clearly and convincingly.

(i) The system must be LTI.

(ii) The system could be LTI, but does not have to be.

(iii) The system cannot be LTI.

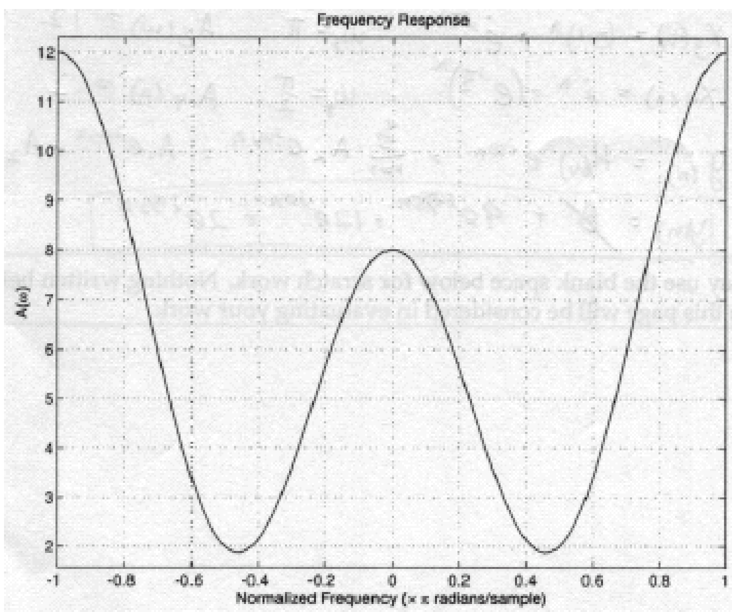
MT2.4 (20 Points) Consider a discrete-time LTI filter $\mathsf{A} : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ having impulse response a and frequency response A. The figure below is a graphical, input-output depiction of the filter:



Recall that the frequency response and impulse response are related as follows:

$$\forall \omega \in \mathbb{R}, \qquad A(\omega) = \sum_{n=-\infty}^{\infty} a(n) e^{-i\omega n}.$$

The figure below depicts $A(\omega), \forall \omega \in [-\pi, +\pi]$. Notice that for this particular filter, $A(\omega)$ is real-valued at all frequencies.



The frequency axis in the figure is normalized by π; hence for example, the normalized frequencies 0.5 and 1 refer to $\omega = \pi/2$ and $\omega = \pi$ radians per sample, respectively.

Determine a reasonable and simple (possibly approximate) expression for the output y of the filter, if the input x is:

$$\forall n \in \mathbb{Z}, \qquad x(n) = e^{i\pi/3} + \cos\left(\frac{4\pi}{5} n\right) + (-1)^n + i^n .$$

Note that there is no "n" in the first term. This is not a typographical error.

MT2.5 (15 Points) The impulse response *h* of a discrete-time LTI system is given by:

$$\forall n \in \mathbb{Z}, \qquad h(n) = \left(\frac{1}{2}\right)^n u(n),$$

where *u* is the unit-step function.

(a) Select the strongest true assertion from the list below.

(i) The system must be causal.

(ii) The system could be causal, but does not have to be.

(iii) The system cannot be causal.

(b) Determine a simple expression for the frequency response *H* of the system. Recall that the frequency response and impulse response are related as follows:

$$\forall \omega \in \mathbb{Z}, \qquad H(\omega) = \sum_{n=-\infty}^{\infty} h(n) e^{-i\omega n}.$$

<u>Hint</u>: You may find the following helpful. If $|\alpha| < 1$, then $\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}$.