

LAST Name \_\_\_\_\_ FIRST Name \_\_\_\_\_

Lab Time \_\_\_\_\_

- **(10 Points)** Print your name and lab time in legible, block lettering above AND on the last page where the grading table appears.
- This exam should take up to 70 minutes to complete. You will be given at least 70 minutes, up to a maximum of 80 minutes, to work on the exam.
- **This exam is closed book.** Collaboration is not permitted. You may not use or access, or cause to be used or accessed, any reference in print or electronic form at any time during the exam, except three double-sided 8.5" × 11" sheets of handwritten notes having no appendage. Computing, communication, and other electronic devices (except dedicated timekeepers) must be turned off. Noncompliance with these or other instructions from the teaching staff—including, for example, commencing work prematurely or continuing beyond the announced stop time—is a serious violation of the Code of Student Conduct. Scratch paper will be provided to you; ask for more if you run out. You may not use your own scratch paper.
- **The exam printout consists of pages numbered 1 through 8.** When you are prompted by the teaching staff to begin work, verify that your copy of the exam is free of printing anomalies and contains all of the eight numbered pages. If you find a defect in your copy, notify the staff immediately.
- Please write neatly and legibly, because *if we can't read it, we can't grade it.*
- For each problem, limit your work to the space provided specifically for that problem. *No other work will be considered in grading your exam. No exceptions.*
- Unless explicitly waived by the specific wording of a problem, you must explain your responses (and reasoning) succinctly, but clearly and convincingly.
- We hope you do a *fantastic* job on this exam.

**MT3.1 (35 Points)** A discrete-time LTI system  $G$  has impulse response  $g$  and frequency response  $G$ . Moreover, we know that the impulse response is real-valued and odd (anti-symmetric). That is,

$$g(n) \in \mathbb{R} \quad \text{and} \quad g(n) = -g(-n) \quad \text{for all } n.$$

(a) (20 Points) Prove that the frequency response  $G(\omega)$  is

- (i) odd (anti-symmetric) with respect  $\omega$ ; and
- (ii) a purely imaginary function of  $\omega$ .

(b) (15 Points) Determine the output  $y$  of the system  $G$  corresponding to an input signal having the following property:

$$x(n+2) = x(n) \quad \text{for all } n.$$

**MT3.2 (70 Points)** The figure below depicts a discrete-time causal LTI system  $G$ .



The frequency response of the system is expressed below:

$$\forall \omega \in \mathbb{R}, \quad G(\omega) = \frac{1 - e^{-i2\omega}}{1 - (0.99)^2 e^{-i2\omega}}.$$

(a) (10 Points) Provide a well-labeled plot of  $|G(\omega)|$ , the magnitude response of the filter. What type of filter is  $G$ ? Explain.

(b) (10 Points) Determine the linear constant-coefficient difference equation that relates the input  $x$  to the output  $y$ .

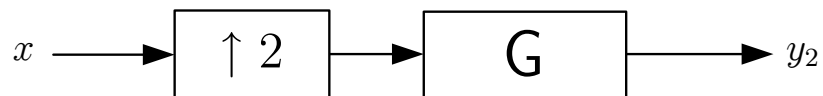
(c) (15 Points) Determine the impulse response  $g$  of the system.

(d) (20 Points) Consider the following input signal:

$$x(n) = 1 + (-1)^n \quad \text{for all } n.$$

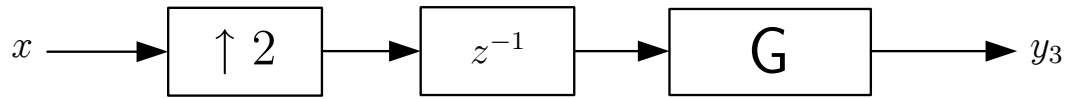
(i) If the input signal  $x$  is applied to the system  $G$ , determine the output signal  $y$ .

(ii) Suppose we apply the input signal  $x$  to the following cascade of systems:



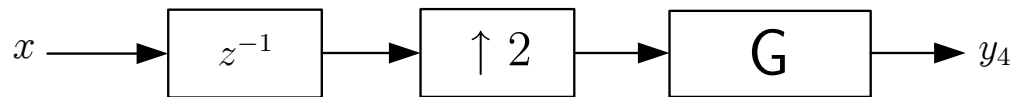
Determine the output signal  $y_2$ .

(iii) Suppose we apply the input signal  $x$  to the following cascade of systems:



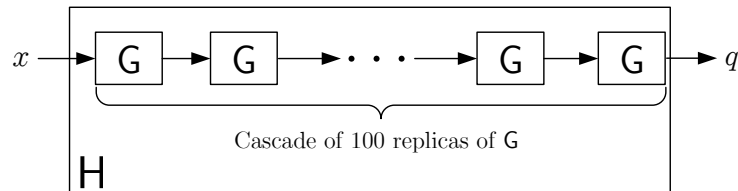
Determine the output signal  $y_3$ .

(iv) Suppose we apply the input signal  $x$  to the following cascade of systems:



Determine the output signal  $y_4$ .

- (e) (15 Points) Consider a system H constructed from the cascade of 100 replicas of the system G, as shown in the diagram below:

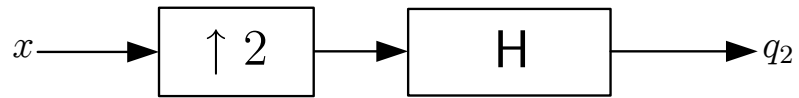


- (i) Determine a reasonably simple expression for  $H(\omega)$ , the frequency response of the system H. Also, provide a well-labeled plot of  $|H(\omega)|$ , the magnitude response of the system H.

- (ii) Determine numerical values for each of the following expressions:

$$\sum_{n=-\infty}^{+\infty} h(n) = \quad \text{and} \quad \sum_{n=-\infty}^{+\infty} (-1)^n h(n) =$$

- (iii) Suppose we apply the input signal  $x$  described described in part (d) to the following cascade of systems:



Determine the output signal  $q_2$ .

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Problem Name	Points	Your Score
1(a)	20	
1(b)	15	
2(a)	10	
2(b)	10	
2(c)	15	
2(d)	20	
2(e)	15	
<b>Total</b>	<b>115</b>	