EECS 20. Midterm No. 1, October 13, 1999.

Please use these sheets for your answer. Add extra pages if necessary and staple them to these sheets. **Write clearly and put a box around your answer, and show your work.**

Print your name below

Last Name _____ First _____

Problem 1:

Problem 2:

Problem 3:

Problem 4:

Total:

1. **20 points** Fill in the blanks:

   (a) If $A = \{1, 2, 3\}$, $B = \{2, 3, *, \#\}$, then $A \cap B =$ and $A \cup B =$

   (b) If the predicates P, Q, R all evaluate to $false$, then $[\neg P \wedge Q] \vee [\neg Q \wedge R] \vee [\neg R \wedge P]$ evaluates to

   (c) If $f : X \rightarrow Y$ and $g : Y \rightarrow Z$, then $g \circ f :$

   (d) Euler's formula is $\exp i\theta =$

   (e) If $A\cos(\omega t + \theta) = \cos(\omega t + \pi/4) + \cos(\omega t - \pi/4)$, then $A =$, $\theta =$

2. **20 points** Determine which of the following functions are periodic and what is their period in seconds or samples.

   (a) $\forall n \in Ints,$ $x(n) = \cos(2\pi n/111).$

   (b) $\forall n \in Ints,$ $x(n) = \cos(2\pi \sqrt{2} n).$

   (c) $\forall t \in Reals,$ $x(t) = \cos(2\pi \sqrt{2} t).$

   (d) $\forall t \in Reals,$ $x(t) = \exp(2\pi 60t + \pi/4).$

3. **30 points** Consider a discrete-time LTI system

$$H : [Ints \rightarrow Comps] \rightarrow [Ints \rightarrow Comps]$$

such that for input signal x, the output signal y is:

$$\forall\ n \in Ints, \quad y(n) = x(n) + x(n-1).$$

(a) What is the output signal y when the input signal is:

$$x(n) = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$

(b) Obtain an expression for the the frequency response $\hat{H}(\omega)$, as a function of ω, the frequency in radians/sample:

$$\forall \omega \in Reals, \quad \hat{H}(\omega) =$$

(c) Obtain expressions for the magnitude and phase response.

(d) Plot the magnitude response $|\hat{H}(\omega)|$ and the phase response $\angle\hat{H}(\omega)$ for $0 \leq \omega < \pi$. (Hint: To plot these first calculate $\hat{H}(\omega)$ for $\omega = 0, \pi/2, \pi$.)
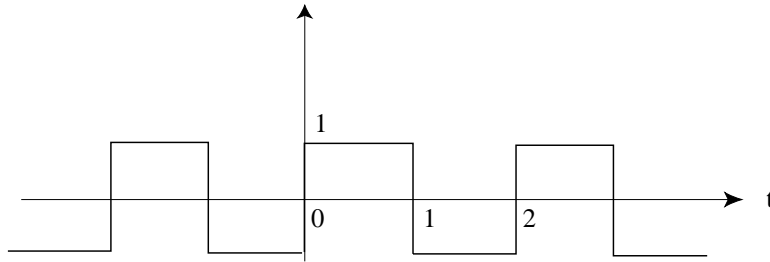
Figure 1: Square wave with period 2 seconds

4. **30 points** The exponential Fourier series of the square wave periodic function x depicted in the figure is of the form:

$$\forall t \in Reals, \quad x(t) = \sum_{k=-\infty}^{\infty} X_k \exp(ik\omega_0 t). \tag{1}$$

(a) What is ω_0?

(b) Calculate the coefficients X_k in (1).