EECS 20. Midterm No. 2
April 16, 2002.

Please use these sheets for your answer and your work. Use the backs if necessary. **Write clearly and put a box around your answer, and show your work.**

Print your name and lab TA's name below

Name: _____

Lab TA: _____

Problem 1:

Problem 2:

Problem 3:

Problem 4:

Total:

1. **30 points.** Consider a continuous-time signal $x : Reals \rightarrow Reals$ defined by

$$\forall t \in Reals, \;\; x(t) = 3 + 2\sin(3t) + 3\cos(4t).$$

(a) Obtain the Fourier series coefficients of $x(t)$, i.e., find the coefficients $A_0, A_1, A_2, \ldots$ and $\phi_1, \phi_2, \ldots$ and w_0 such that

$$x(t) = A_0 + \sum_{k=1}^{\infty} A_k \cos(kw_0 t + \phi_k).$$

(b) Obtain the Fourier series expansion for $x(t)$, i.e., find the coefficients X_k for all $k \in Integers$ such that

$$x(t) = \sum_{k=-\infty}^{\infty} X_k \, e^{ikw_0 t} \quad .$$

(c) Consider a continuous-time LTI system $Filter : [Reals \rightarrow Reals] \rightarrow [Reals \rightarrow Reals]$ with the following frequency response:

$$H(w) = 2, \quad |w| \geq 2,$$
$$H(w) = 0, \quad |w| < 2.$$

Such a filter is an amplifying high-pass filter. Give a simple expression for the output $y(t)$ of the system, where $y = Filter(x)$.

2. **20 points.** Consider a system whose input and output are related by

$$\forall n \in Integers,\ \ y(n) = 2\,x(n-2) + 1.1\,y(n-1).$$

(a) Construct a state-space model for the system. It is sufficient to give the state definition, the A matrix, vectors b and c, and scalar d.

(b) Give an expression for the zero-state impulse response.

(c) Recall that a system is stable if a bounded input always produces a bounded output. Is this system stable? Explain.

3. **25 points.** Consider discrete-time systems with input $x : Integers \rightarrow Reals$ and output $y : Integers \rightarrow Reals$. Each of the following defines such a system. For each of the following, indicate whether it is linear only (L), time-invariant only (TI), both (LTI), or neither (N). Note that no partial credit will be given for these questions.

   (a) $\forall n \in Integers,\ \ y(n) = x^3(n-10) = (x(n-10))^3$

   (b) $\forall n \in Integers,\ \ y(n) = \cos(x(n))$

   (c) $\forall n \in Integers,\ \ y(n) = n$

   (d) $\forall n \in Integers,\ \ y(n) = \max\{|x(n)|,\ |x(n-1)|\}$

   (e) $\forall n \in Integers,\ \ y(n) = x(-n)$

4. **15 points.** Consider a continuous-time LTI system S. Suppose that when the input is given by

$$x(t) = \begin{cases} \sin(\pi t) & 0 \le t < 1 \\ 0 & \text{otherwise} \end{cases}$$

then the output $y = S(x)$ is given by

$$y(t) = \begin{cases} \sin(\pi t) & 0 \le t < 1 \\ \sin(\pi(t-2)) & 2 \le t < 3 \\ 0 & \text{otherwise} \end{cases}$$

for all $t \in$ *Reals*.

(a) Carefully sketch these two signals.

(b) Give a simple expression and a sketch for the output of the same system if the input is

$$x(t) = \begin{cases} \sin(\pi t) & 0 \le t < 1 \\ -\sin(\pi(t-1)) & 1 \le t < 2 \\ 0 & \text{otherwise} \end{cases} .$$

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