EECS 20. Midterm No. 1
February 27, 2004. Solution

1. **30 points. 5 points for each part.** Please indicate whether the following statements are true or false. There will be no partial credit. They are either true or false. So please be sure of your answer.

   (a) $\forall\, n \in \textit{Integers}, \quad (n, n+1, \sqrt{n}) \in \textit{Integers}^3$.
   False

   (b) If A contains 6 elements, $P(A)$ contains 64 elements. $P(A)$ denotes the power set of A.
   True

   (c) The function *square*: $[0, \infty) \to [0, \infty)$, given by $\forall x, f(x) = x^2$, is one-to-one and onto.
   True

   (d) Consider a deterministic state machine with 5 states, $\textit{Inputs} = \{0, 1\}$, $\textit{Outputs} = \{0, 1\}$. Suppose the signal $y : \textit{Nats}_0 \to \{0, 1\}$ is the output response to the input signal $x = (0, 0, 0, ...)$ (all zeros). Then y is eventually periodic, i.e., there is an integer $1 \leq p \leq 5$ such that

   $$\forall n \geq 4, \quad y(n) = y(n+p).$$

   True
   Proof. Let $s(0), s(1), \cdots$ be the state response to the input signal x. Because $s(0), \cdots, s(5)$ can take at most 5 distinct values, there must be a repetition, i.e. there must exist $0 \leq k \leq 4, 1 \leq p$ with $k + p \leq 5$ such that $s(k) = s(k+p)$.

   (e) Suppose state machine A simulates state machine B, A is deterministic and B is possibly non-deterministic. Then B simulates A.
   True
   Proof. Suppose $S \subset \textit{States}_B \times \textit{States}_A$ is the simulation relation showing that A simulates B. To show that B simulates A use the relation $S' = \{(s_A, s_B) \mid (s_B, s_A) \in S\}$.

   (f) Consider two state machines A and B with state spaces $\textit{States}_A$ and $\textit{States}_B$. If in each state machine, all states are reachable, then in the cascade composition, all states in $\textit{States}_A \times \textit{States}_B$ are reachable.
   False

2. **10 points** The following Matlab program plots the graph of the function y.

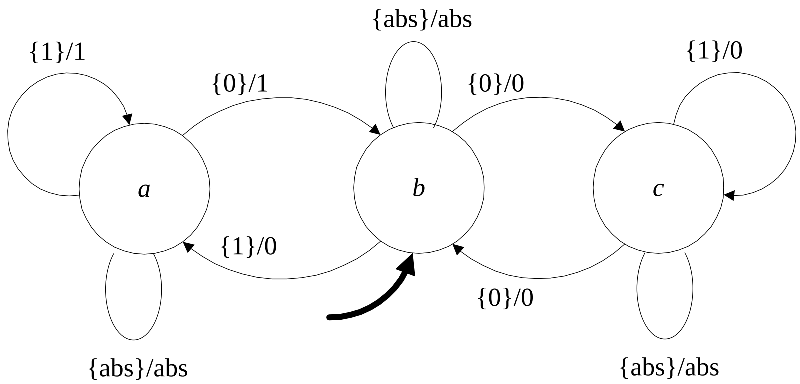
```
denseTime = [-1:0.01:1];
y = exp(-denseTime).*sin(10*pi*denseTime);
plot(denseTime,y);
```

(a) Domain of y = $\boxed{\{-1, -0.99, \cdots, 0.99, 1.0\}}$

(b) Range of y = $\boxed{\textit{Reals}}$. In fact, any set R with $\{e^{-x} \sin x \mid x \in \textit{Domain}(y)\} \subset R$ will serve as range of y.

(c) Provide a mathematical expression:
$\forall x \in$ *Domain of y*, $\quad y(x) = \boxed{e^{-x} \sin(10\pi x)}$

3. **25 points. 2 points for parts (a)-(h), (j), 7 points for (i).** Consider the state transition diagram shown below.



(a) Add the transitions corresponding to the input symbol *absent* and give each of the following:

(b) *States* = $\{a, b, c\}$

(c) *Inputs* = {*1, 0, absent*}

(d) *Outputs* = {*1, 0, absent*}

(e) *OutputSignals* = $[Nats_0 \to \{1, 0, absent\}]$

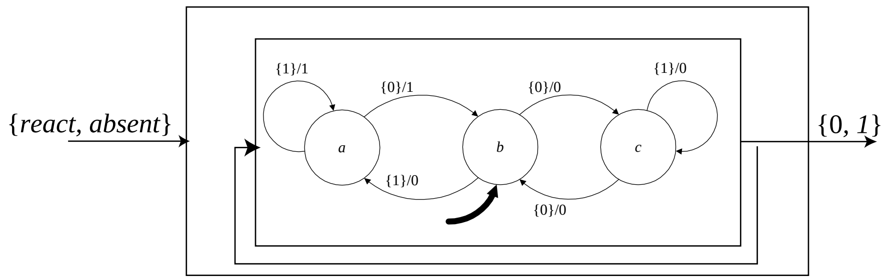
(f) Give the domain and range of the *update* function.
Its domain is *States* × *Inputs* and its range is *States* × *Outputs*.

(g) Fill in the table for *update*:

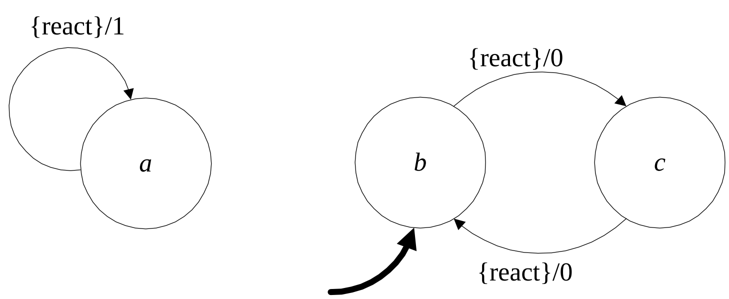
current state	(*next state, output symbol*) under specified input symbol		
	1	0	*absent*
a	$(a, 1)$	$(b, 1)$	$(a, absent)$
b	$(a, 0)$	$(c, 0)$	$(b, absent)$
c	$(c, 0)$	$(b, 0)$	$(c, absent)$

(h) *initialState* = $\boxed{b}$

(i) Compose this state machine in a feedback loop, where its output is connected to its input as in the figure below. Assume the output of the composition is the output of this state machine. Draw the state transition diagram for the composition, taking as its input alphabet the set $\{react, absent\}$.



The composed machine is shown below:



(j) Which states can be reached from the initial state in the feedback machine?
The reachable states are $\boxed{\{b, c\}}$.

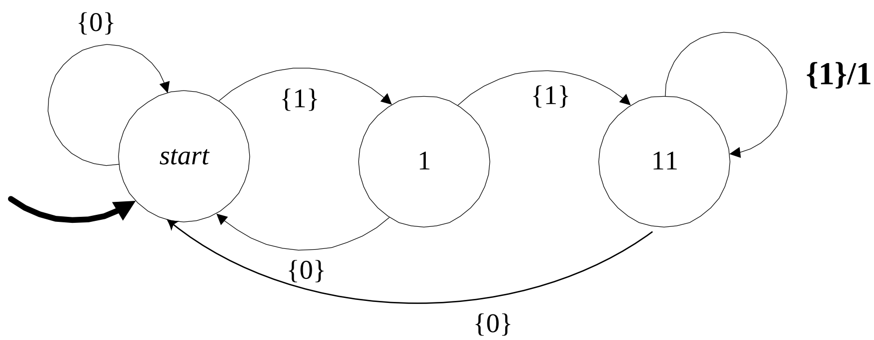
4. **15 points. 8 points for (a), 7 points for (b)**

(a) Design a state machine M, with *Inputs* $=$ *Outputs* $= \{0, 1, \textit{absent}\}$, that has three states, and whose input-output function F is given by (neglecting the stuttering input *absent*):

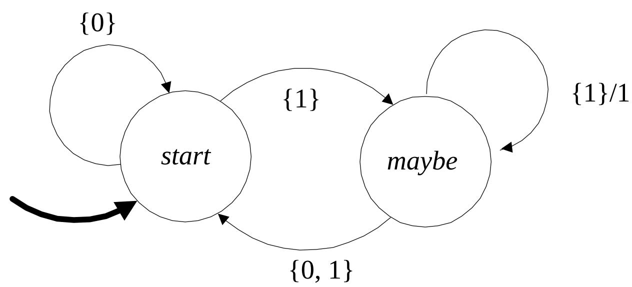
$\forall x \in \textit{InputSignals}, \forall n \in \textit{Nats}_0$,

$$F(x)(n) = \begin{cases} 1, & \text{if } (x(n-2), x(n-1), x(n)) = (1,1,1) \\ \textit{absent}, & \text{else} \end{cases}$$

The machine shown below is the design M



(b) Now consider the non-deterministic machine N below.



Determine whether N simulates your machine M and write down the relevant simulation relation, if any.

$\boxed{\text{No}}$. To see this, observe that if there is a simulation relation it must be

$$S = (\textit{start}, \textit{start}), (1, \textit{maybe}), (11, \textit{maybe})\}.$$

The response of the 3-state machine to the input sequence 1,1,$\cdots$ is $\textit{absent}, \textit{absent}, \cdots$ and the state response is $\textit{start}, 1, 11, \cdots$; but the response of the N is $\textit{absent}, \textit{absent}, \cdots$ and the state response is $\textit{start}, \textit{maybe}, \textit{start}, \cdots$. But $(11, \textit{start}) \notin S$.

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