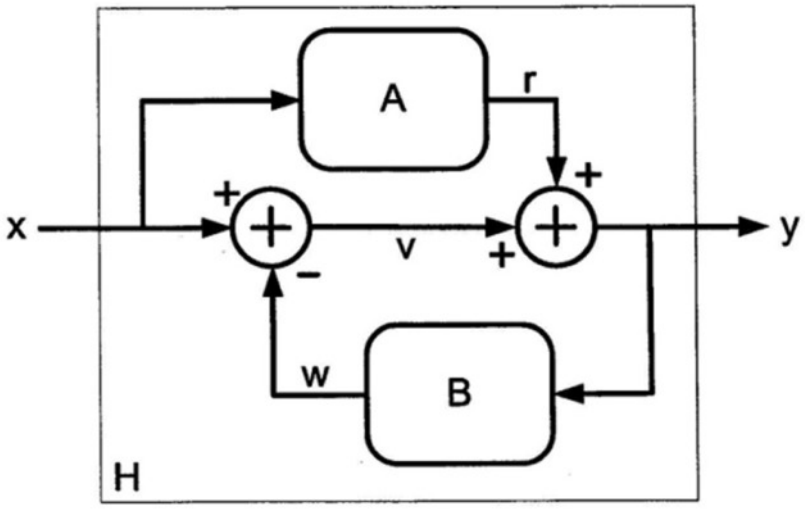
EECS 20N, Spring, 2007, Midterm 2, Ayazifar

MT2.1 (15 Points) Consider a well-structured interconnection of H of discrete-time LTI systems A and B, as shown in the figure below. Each of the individual systems is a function defined on $[\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$.



Let the respective frequency responses be A and B, defined on $\mathbb{R} \to \mathbb{C}$.

Determine the composite system's frequency response $H : \mathbb{R} \to \mathbb{C}$ in terms of the frequency responses A and B of the individual components. Reduce your expression to the simplest form possible.

Note: The intermediate signals r, v, and w have been labeled on the diagram for your convenience. It is not necessary that you make use of them in your work.

MT2.2 (25 Points) The following discrete-time systems F, G, and H should be treated mutually independently; properties that hold for one system cannot be *assumed* to hold for the others.

For each part, explain your reasoning succinctly, but clearly and convincingly.

(a) (10 Points) A discrete-time system $\mathrm{F} : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ produces the output signal y,

$$y(n) = \cos(\frac{\pi}{4}n), \quad \forall n,$$

in response to the input signal x,

$$x(n) = e^{i\pi n/4}, \quad \forall n.$$

Select the strongest true assertion from the list below.

(i) The system must be LTI.

(ii) The system could be LTI, but does not have to be.

(iii) The system cannot be LTI.

If your choice is (i) or (ii), please answer the following:

(I) Provide as much information about the frequency response of the (or an) LTI system consistent with the input-output pair of signals x and y. In particular, specify all inferable values of the frequency response $F(\omega), \omega \in \mathbb{R}$.

(II) Could the impulse response f of the system be real-valued? Explain your reasoning succinctly, but clearly and convincingly.

(b) (6 Points) A discrete-time system G : $[\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ produces the output signal y,

$$y(n) = e^{i\pi n/4}, \quad \forall n,$$

in response to the input signal x,

$$x(n) = \cos(\frac{\pi}{4}n), \quad \forall n.$$

Select the strongest true assertion from the list below.

(i) The system must be LTI.

(ii) The system could be LTI, but does not have to be.

(iii) The system cannot be LTI.

(c) (9 Points) A discrete-time system H : $[\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ produces the output signal y,

$$y(n) = \cos(\frac{\pi}{4}n), \quad \forall n,$$

in response to the input signal x,

$$x(n) = \sin(\frac{\pi}{4}n), \quad \forall n.$$

Select the strongest true assertion from the list below.

(i) The system must be *memoryless*.

(ii) The system could be *memoryless*, but does not have to be.

(iii) The system cannot be *memoryless*.

MT2.3 (35 Points) Consider a discrete-time LTI system $\mathrm{F} : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ having input signal x and output signal y, as shown below:

If the input signal is the one-sided decaying exponential

$$x(n) = \alpha^n u(n), \quad \forall n,$$

where $0 < |\alpha| < 1$, the output signal is simply the Knonecker delta function, i.e.,

$$y(n) = \delta(n), \quad \forall n.$$

(a) (10 Points) Determine a simple expression for the frequency response values $F(\omega), -\pi \leq \omega \leq +\pi$.

Hint: You may find the following helpful. If $|\beta| < 1$,then $\sum_{n=0}^{\infty} \beta^n = \frac{1}{1-\beta}$.

(b) (10 Points) Determine a simple expression for $f(n), \forall n$, where f is the impulse response of the system F.

Note that it is possible to determine the impulse response f without knowing the frequency response F.

(c) (7 Points) Select the strongest true assertion from the list below.

(i) The system must be *memoryless*.

(ii) The system could be *memoryless*, but does not have to be.

(iii) The system cannot be *memoryless*.

(d) (8 Points) A discrete-time anti-causal system is defined as a system whose instantaneous output does *not* depend on past values of its input.

That is, a system is said to be *anti-causal* if, and only if, the instantaneous output $y(n)$ depends *at most* on the input values $x(m), m \geq n$.

Select the strongest true assertion from the list below.

(i) The system must be *anti-causal*.

(ii) The system could be *anti-causal*, but does not have to be.

(iii) The system cannot be *anti-causal*.

MT2.4 (30 Points) Consider a discrete-time LTI system $\mathrm{F} : [\mathbb{Z} \to \mathbb{C}] \to [\mathbb{Z} \to \mathbb{C}]$ having input signal x and output signal y, as shown below:

The frequency response $F : \mathbb{R} \to \mathbb{C}$ is given by

$$F(\omega) = \frac{1 + e^{-i2\omega}}{1 + (0.99)^2 e^{-i2\omega}}, \quad -\pi \leq \omega \leq +\pi.$$

(a) (10 Points) Provide a well-labeled sketch of the magnitude response $|F(\omega)|$, $-\pi \leq \omega + \pi$.
What type of filter is F: low-pass, band-pass, high-pass, all-pass, or notch?
Possibly helpful: $(0.99)^2 \approx 0.98$.

(b) (10 Points) Suppose the input to the system is described by

$$x(n) = 1 + 2e^{i\pi n/4} + 3\cos(\frac{\pi}{2}n) + 4(-1)^n, \quad \forall n.$$

Determine the output values $y(n), \forall n$.

(c) (10 Points) Determine the linear, constant-coefficient difference equation that governs the input-output behavior of the system.