UNIVERSITY OF CALIFORNIA
College of Engineering
Department of Electrical Engineering and Computer Sciences

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Thursday, March 11, 2010 | **SPRING 2010**

You should write your results on the exam sheets only. Partial credit will be given only if you show your work and reasoning clearly.

Throughout the exam, you can ignore flicker noise, assume that the r_o of the transistors is infinite, and ignore all capacitors except those drawn in the circuit unless the problem states otherwise.

Name: Solutions

SID: ______________________

Problem 1	______/ 10
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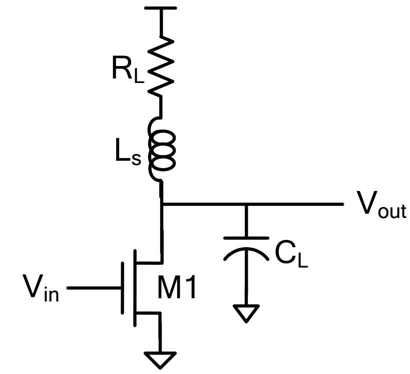
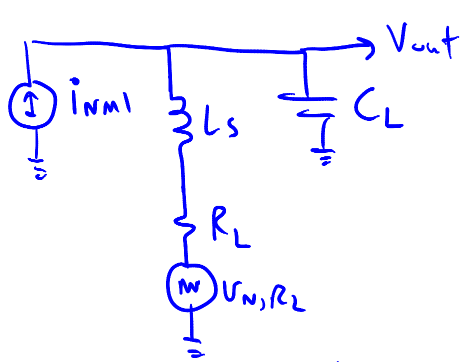
Total	______/ 37

Problem 1 (10 points) Noise



What is the total voltage noise variance v_{on}^2 at the output of the amplifier shown above? You should provide your final answer in terms of k, T, γ, R_L, L_s, C_L, and $A_{v0} = g_m R_L$.

Small signal model:



M1 $\rightarrow$

$$\frac{V_{out}(s)}{i_{nM1}(s)} = (R_L + sL_s) \,||\, 1/sC_L$$

$$= \frac{R_L + sL_s}{s^2 L_s C_L + sR_L C_L + 1}$$

$$= R_L \cdot \frac{1 + sL_s/R_L}{s^2 L_s C_L + sR_L C_L + 1}$$

$$\int_0^\infty \left|\left| \frac{V_{out}(f)}{i_{nM1}(f)} \right|\right|^2 df = \frac{R_L^2}{4R_L C_L} \cdot \left(\frac{L_s^2/R_L^2}{L_s C_L} + 1 \right)$$

$$= \frac{R_L}{4C_L} \cdot \left(\frac{L_s/R_L}{R_L C_L} + 1 \right)$$

So: $$\overline{v_{on,M1}^2} = 4kT\gamma g_m \cdot \frac{R_L}{4C_L} \left(\frac{L_s/R_L}{R_L C_L} + 1 \right)$$

$$= \frac{kT}{C_L} \cdot \gamma A_{vo} \left(\frac{L_s/R_L}{R_L C_L} + 1 \right)$$

R_L $\swarrow$

$$\frac{V_{out}(s)}{V_{n,R_L}(s)} = \frac{1/sC_L}{1/sC_L + R_L + sL_s}$$

$$= \frac{1}{s^2 L_s C_L + sR_L C_L + 1}$$

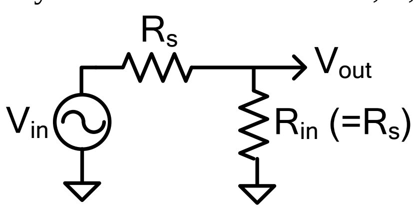
So: $$\overline{v_{on,R_L}^2} = 4kTR_L \cdot \frac{1}{4R_L C_L} = \frac{kT}{C_L}$$

$$\boxed{\overline{v_{on,tot}^2} = \frac{kT}{C_L} \left[1 + \gamma A_{vo} \left(\frac{L_s/R_L}{R_L C_L} + 1 \right) \right]}$$

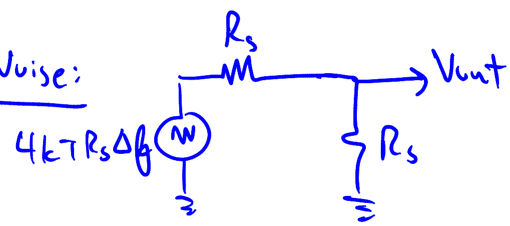
Problem 2 (18 points) Termination and Noise

As we will see later on in class, in many applications we have to design our amplifiers such that they provide an input termination resistance that is equal to the resistance of the source (e.g., in order to avoid reflections). In this problem we will examine a few options for implementing the amplifier and the termination resistor.

a) **(4 pts)** For the circuit shown below, what is the noise voltage density ($v_{on}^2/\Delta f$) at the output if you assume that the source resistance R_s is noiseless (i.e., noise is only contributed by R_{in})? Under this same condition and assuming that V_{in} is a sinusoid with amplitude of A_{in}, what is the SNR of V_{out}? You should provide your answers in terms of k, T, R_s, Δf, and A_{in}.

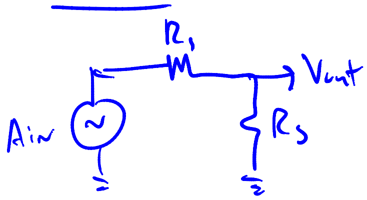


Noise:



$$\boxed{\frac{v_{on}^2}{\Delta f} = \left(\frac{1}{2}\right)^2 \cdot 4kTR_s = kTR_s}$$

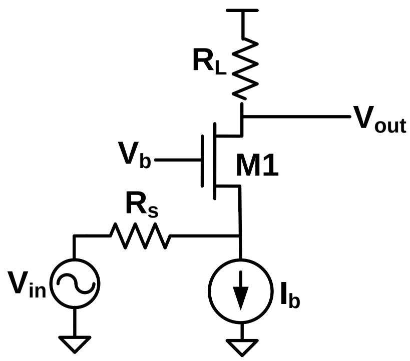
Signal:



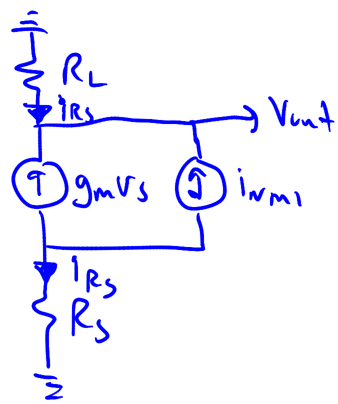
$$\overline{V_{out}^2} = \frac{A_{in}^2}{2} \cdot \left(\frac{1}{2}\right)^2 = \frac{A_{in}^2}{8}$$

$$\boxed{SNR = \frac{A_{in}^2}{8kTR_s\Delta f}}$$

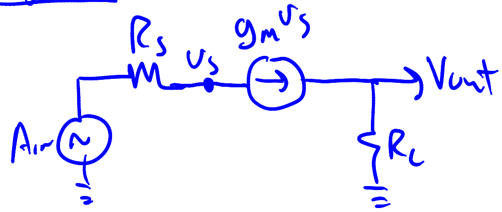
b) **(6 pts)** Your colleague Ace says he thinks he can do better with an active circuit to create the termination and amplify the signal. Assuming that R_s and R_L are noiseless and that $1/g_{m1} = R_s$ (i.e., the transistor provides the termination resistance) and still with an input sinusoid whose amplitude is A_{in}, now what is the SNR at V_{out}? You can assume that $\gamma = 1$, and you should provide your answers in terms of k, T, R_s, Δf, and A_{in}.



Noise:



Signal:



$$v_s = \frac{1}{1+g_m R_s} \cdot V_{in}$$

$$V_{out} = \frac{1}{2} \cdot V_{in} \cdot g_m \cdot R_L$$

$$\overline{V_{out}^2} = \frac{A_{in}^2}{2} \cdot \frac{g_m^2 \cdot R_L^2}{4}$$

$$i_{R_s} = \frac{1/g_m}{1/g_m + R_s} \cdot i_{nM1} = \frac{1}{1+g_m R_s} \cdot i_{nM1}$$

$$g_m R_s = 1 \rightarrow i_{R_s} = \frac{1}{2} \cdot i_{nM1}$$

$$\frac{\overline{v_{on}^2}}{\Delta f} = \frac{1}{4} \cdot 4kT\gamma g_m \cdot R_L^2$$

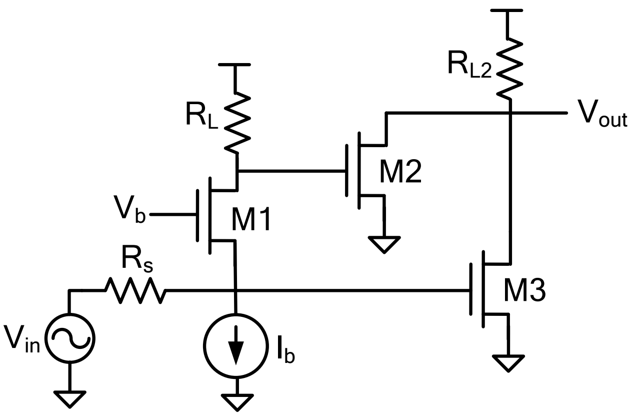
$$= kT/R_s \cdot R_L^2 \qquad (\gamma = 1)$$

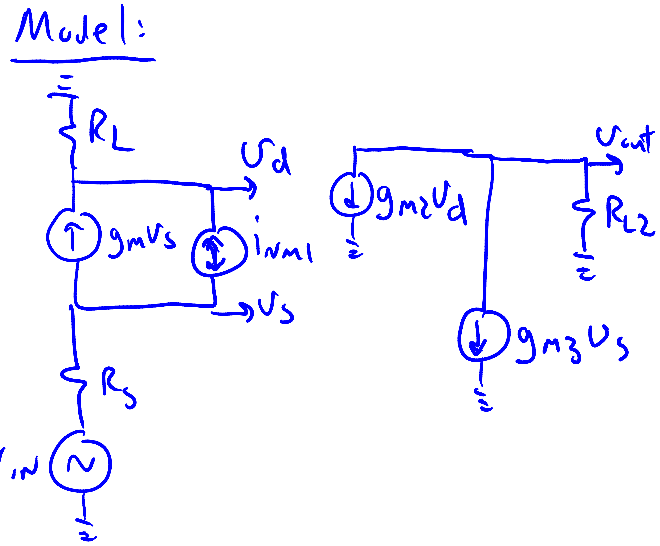
$$SNR = \frac{A_{in}^2 g_m^2 R_L^2}{8kTR_L^2 \Delta f / R_s}$$

$$SNR = \frac{A_{in}^2 g_m^2 R_s}{8kT\Delta f} \longrightarrow \boxed{SNR = \frac{A_{in}^2}{8kTR_s \Delta f}}$$

(Same as before)

c) **(8 pts)** Your other colleague Pat suggests she can achieve better noise performance with the circuit shown below. Ignoring all noise sources except for the noise from M1 and assuming that $g_{m2}R_L = g_{m3}R_s$ and that $1/g_{m1} = R_s$, what is the noise voltage density at V_{out}? What is the small signal gain of the circuit from V_{in} to V_{out}? You should provide your answers in terms of k, T, R_s, R_L, R_{L2}, g_{m3}, and Δf.





Noise:

$$v_s = i_{nM1}\cdot\frac{R_s}{1+g_{m1}R_s} = i_{nM1}\cdot\frac{R_s}{2}$$

$$v_d = -i_{nM1}\cdot\frac{R_L}{1+g_{m1}R_s} = -i_{nM1}\cdot\frac{R_L}{2}$$

$$v_{out} = \left(-g_{m3}i_{nM1}\frac{R_s}{2} + g_{m2}i_{nM1}\frac{R_L}{2}\right)\cdot R_{L2}$$

$$v_{out} = \frac{i_{nM1}}{2}\cdot\left(g_{m2}R_L - g_{m3}R_s\right)\cdot R_{L2}$$

$$g_{m2}R_L = g_{m3}R_s$$

$$\hookrightarrow \boxed{v_{on}^2 = 0}$$

Signal:

$$v_s = V_{in}/2$$

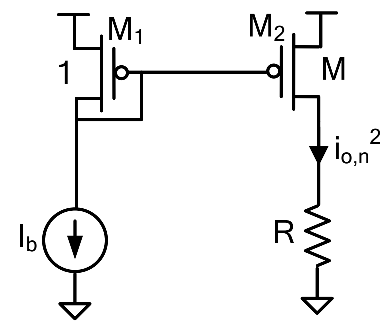
$$v_d = g_{m1}R_L\cdot\frac{V_{in}}{2}$$

$$V_{out} = -g_{m3}R_{L2}\frac{V_{in}}{2} - g_{m2}R_{L2}g_{m1}R_L\cdot\frac{V_{in}}{2} \qquad (g_{m1} = 1/R_s)$$

$$\frac{V_{out}}{V_{in}} = -\frac{1}{2}\cdot\left(g_{m3}R_{L2} + \frac{g_{m2}R_L R_{L2}}{R_s}\right) = -\frac{1}{2}\cdot\frac{R_{L2}}{R_s}\left(g_{m3}R_s + g_{m2}R_L\right)$$

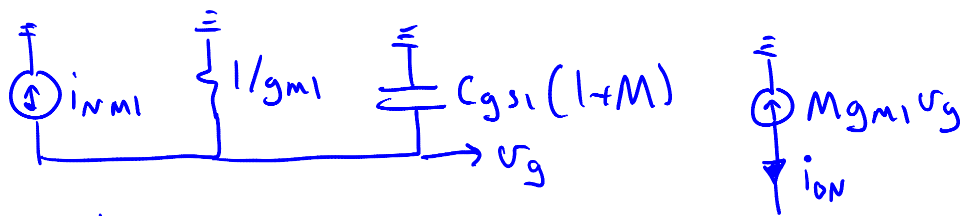
$$\boxed{\frac{V_{out}}{V_{in}} = -g_{m3}R_{L2}}$$

Problem 3 (9 points) Current Mirror Issues



d) **(4 pts)** For the circuit shown above, what is the total noise current flowing into R (i_{on}^2) due to the noise current of device M1? You can ignore all capacitors except for the C_{gs} of the PMOS transistors, and you should provide your answer in terms of k, T, γ, g_{m1}, C_{gs1}, and M.

small signal

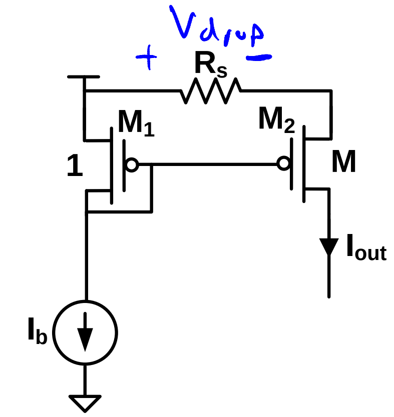


$$\frac{v_g(s)}{i_{nm1}(s)} = \frac{1/g_{m1}}{1 + s\frac{(1+M)C_{gs1}}{g_{m1}}}$$

$$v_{n,g}^2 = 4kT\gamma g_{m1}\cdot\frac{1}{g_{m1}^2}\cdot\frac{g_{m1}}{4(1+M)C_{gs1}} = \frac{kT}{C_{gs1}}\cdot\frac{\gamma}{(1+M)}$$

$$i_{on}^2 = M^2 g_{m1}^2 \cdot v_{n,g}^2$$

$$\boxed{i_{on}^2 = kT\gamma g_{m1}\cdot\frac{M^2}{(M+1)}\cdot\frac{g_{m1}}{C_{gs1}}}$$



e) **(5 pts)** Now let's look at why we want to route currents instead of voltages in our mirrors. For the circuit shown above, approximately what is the error in output current (I_{out}) due to R_s? You should provide your answer as a function of V_1*, M, I_b, and R_s.

* Assume that error in I_{out} is small compared to ideal value:

$$V_{drop} = M I_b \cdot R_s$$

* If drop is small, can treat this as a small signal input into M_2:

$$\Delta I_{out} = -M g_{m1} \cdot V_{drop} = -M^2 g_{m1} I_b \cdot R_s$$

$$g_{m1} = \frac{2 I_b}{V_1^*}$$

$$\boxed{\Delta I_{out} = -\frac{2 M I_b R_s}{V^*} \cdot M \cdot I_b}$$