CS-174 Final May 19, 1992
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There are 6 problems marked **(E)** , and 4 problems **(H)** . Each question is 10 points, but your two highest scores on a **(H)** question are doubled. (It is possible to score 120 points.) 90 points is enough for an A on the exam, so a student who gets two **(H)** questions and 6 of the remaining 8 question has an A with 10 points to spare.

1. **(E)** Two six sided dice are rolled. For each pair of events in the following table, determine if they are independent and/or disjoint.

Event A	Event B	Independent?	Disjoint?
First die comes up 3	First die comes up 3 or 4	No	No
First die comes up 6	First die comes up 1 or 2		
First die comes up 6	Second die comes up 1 or 2		
First die comes up 5	Dice add to 6		
First die comes up 5	Dice add to 7		
First die comes up 5	Dice add to 12		
First die comes up 5	Dice add to 13		

2. **(E)** Prove that all planar embeddings of a given connected planar graph have the same number of faces.

3. **(E)** A 5 card hand is dealt from a standard 52 card deck. Let the events

$$Q = \text{“The hand contains at least one Queen.”}$$
$$H = \text{“The hand contains at least one Heart.”}$$

Calculate $\mathbf{P}\{Q\}$, $\mathbf{P}\{H\}$, $\mathbf{P}\{Q \vee H\}$ and $\mathbf{P}\{Q \wedge H\}$. (Be sure to calculate the easier of $\mathbf{P}\{Q \vee H\}$ and $\mathbf{P}\{Q \wedge H\}$ first!)

4. **(E)** How many 4-digit campus telephone numbers have one or more consecutive repeated digits? (Each digit is randomly selected from $\{0, 1, \ldots, 9\}$. 4422 counts, but 2424 doesn't.)

5. **(E)** A tree has $6k$ nodes,

- $2k$ nodes of degree 1
- $3k$ nodes of degree 2
- k nodes of degree 3

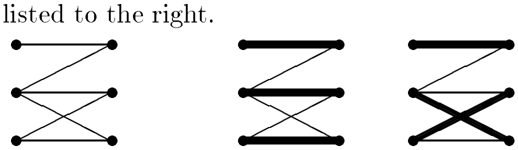
Find k and show that it is uniquely determined.

6. **(E)** An ASCII character is 8 bits. Suppose each character is transmitted along a modem with an extra parity bit which is the exclusive-or of the 8 bits.

(a) Describe the set C of 9-bit code words transmitted.
(b) Find the hamming distance, d, of C.
(c) How many errors can be detected in the code?
(d) How many errors can be corrected in the code?

7. **(H)**

Let G be a random $n \times n$ bipartite graph with each edge included independently with probability $\frac{1}{n}$. Let N be the number of ways to make a perfect matching in G. For example, if G is the following graph, $N = 2$, and the two perfect matchings are listed to the right.



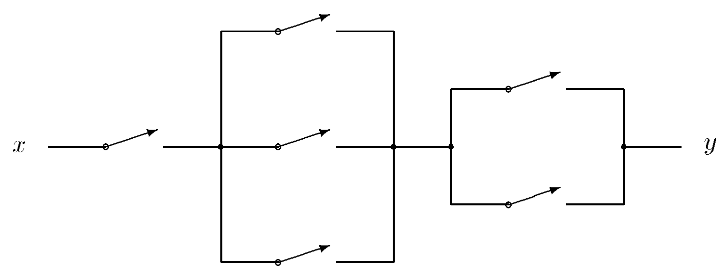
- (7 points) What is $\mathbf{E}\{N\}$?
- (3 points) How does $\mathbf{E}\{N\}$ compare with $\mathbf{P}\{N \geq 1\}$? What does this say about the probability G has a perfect matching when $n \to \infty$?

8. **(H)** A tournament is a directed graph with exactly one edge between every pair of vertices. In other words, to get a tournament, take a complete undirected graph and direct each edge. Show that every tournament has a hamiltonian path.

   Hint: One way to begin a proof is:

   > Let v be any vertex in tournament G. Partition the vertices of G into three sets, $\{v\}$, S, and T, where S is the set of vertices in G which point to v, and T is the set of vertices which v points to.

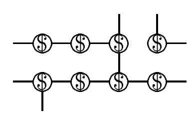
9. **(H)** Assume each switch in the following circuit will be closed (i.e., a connection is made) independently with probability p.



   (a) Find the probability that all switches are closed.
   (b) Find the probability that x and y are connected.
   (c) You do a test and find that x and y are connected. Now what is the probability that all switches are closed?

10. **(H)**

   (a) Find all winning moves in the following Nimstring position.



   (b) Draw the corresponding Dots & Boxes position. How many boxes will you get in a well played game from this position?