CS 70 Discrete Mathematics for CS

Spring 2002 Vazirani Midterm 1

PRINT your name: ____________________________SID________________________

This is a CLOSED BOOK examination. Do all your work on the pages of this examination. Give reasons for all your answers.

1. (15 pts.) Satisfiability and all that

For each of the following Boolean expressions, decide if it is (i) valid (ii) satisfiable (iii) unsatisfiable. (Give *all* applicable properties.)

(a) (10) [(P ===> Q) ^ (Q ===> R)] ===> (P ===> R)

(b) (5) ~(P ===> Q) ^ ~(Q ===> P)

2. (Each 10 pts.) Logic and Proofs

(a) Can you define *open sentences* (i.e., sentences whose truth value depends on some variable x) P(x) and Q(x) and a universe U so that

(for all x in U)(P(x) ==> Q(x)) is false, and

(for all x in U)(Q(x) ==> P(x)) is false?

If yes, give an example. If no, explain why not.

(b) Write a *DNF* formula that expresses the constraint that at least two of

X1, X2, X3, X4 are true.

(c) Prove that for all x in the set of real numbers, if sqrt(2) + x is rational, then x is irrational. What proof technique did you use?

3. (15 pts.) Induction: For every *n* in N let P(n) be a statement about *n*. Suppose that P(13) is false, and for every *n* in N, P(n) ==> P(n + 1). What can we

conclude about P(1)? Why?

4. (10 pts.) Proof to Grade

What is wrong with the following induction proof?

Claim: (for all *n* in N)(*n*^2 <= *n*) (<= means less than or equal)

Proof:

(i) Base Case: When *n* = 1, the statement is 1^2 <= 1 which is true.

(ii) Inductive step: Now suppose that k is in N, and k^2 <= k.

We need to show that (k +1)^2 <= k + 1

Working backwards we see that:

(k +1)^2 <= k + 1

k^2 + 2k + 1 <= k + 1

k^2 + 2k <= k

k^2 <= k

So we get back to our original hypothesis which is assumed to be true. Hence, for every *n* in N we know that *n*^2 <= *n*.

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5. (20 pts.) Answer exactly one of the following:

Fibonacci numbers Recall that the Fibonacci numbers are defined by F(0) = 0, F(1) = 1 and for all n >= 2, F(n) = F(n 1) + F(n 2). Prove by induction that the sum from I=k to n of F(I) = F(n + 2) F(k + 1).

OR

Stable Marriage Consider the asymmetric situation where there are n + 1 boys and n girls (each with their preference lists as before). Does the TMA (the algorithm presented in lecture) always find a stable pairing that matches n of the boys with n of the girls? Justify your answer.

Hint: consider an n + 1-st virtual girl. Where in each boys preference list would you place her?

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