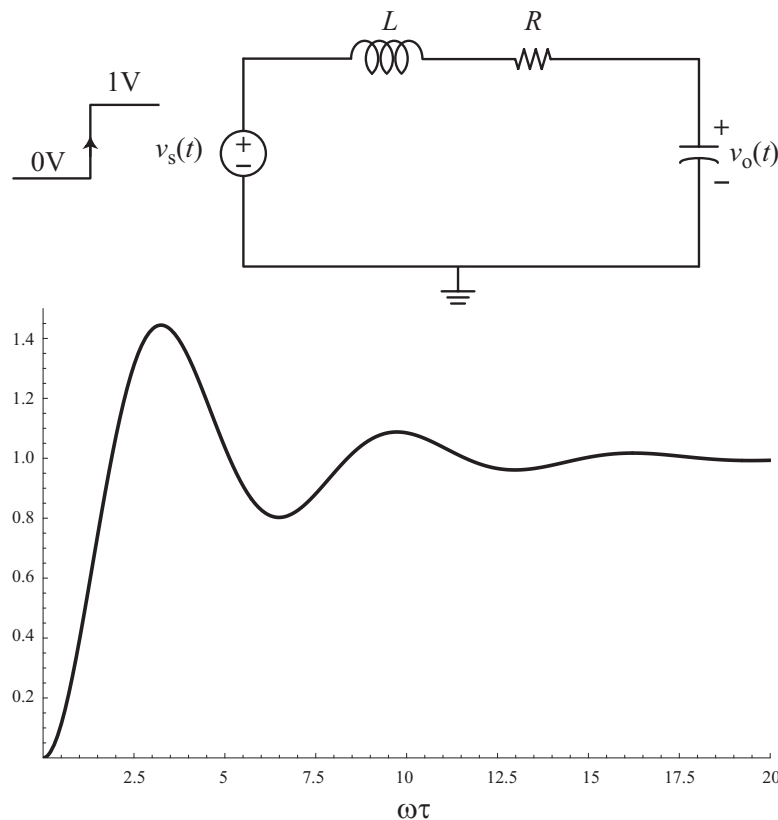


Midterm Exam (closed book)  
Thursday, September 25, 2003

Helpful hints appear at the end of the exam.

1. (20 points) The step response of the series LCR circuit is shown below.



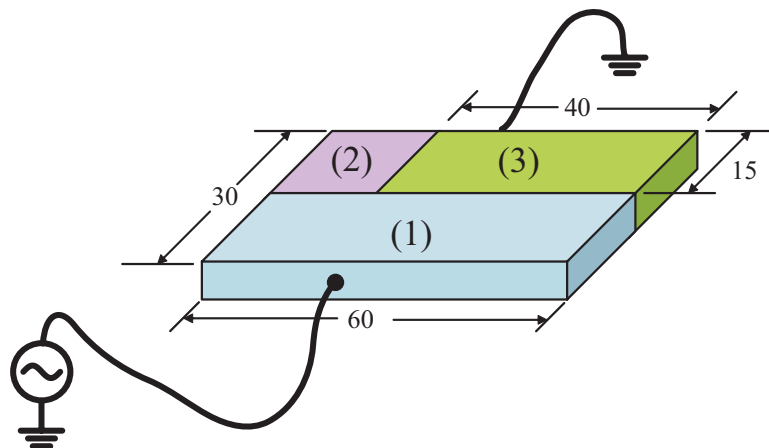
- (a) Is the damping factor  $\zeta$  greater than or less than unity?

- (b) Qualitatively, describe the effect of decreasing the capacitance of the circuit.

(c) If it is desired to obtain a critically damped response, find the value of the series resistance  $R$  if  $\sqrt{L/C} = 50\Omega$ .

(d) If the circuit is now driven with a sinusoidal voltage source of frequency  $\omega \neq \omega_0$ , find an expression for the power dissipation in the resistor  $R$ ?

2. (20 points) For the resistor shown below



The sheet resistances of the region are  $R_{\square}^1 = 10\text{k}\Omega/\text{sq}$ ,  $R_{\square}^2 = 50\text{k}\Omega/\text{sq}$ ,  $R_{\square}^3 = 30\text{k}\Omega/\text{sq}$ .

(a) Calculate the resistance of the structure



(b) At what voltage level does Ohm's law begin to fail, assuming that the carriers saturate when the electric field approaches  $10^4\text{V/cm}$ ?



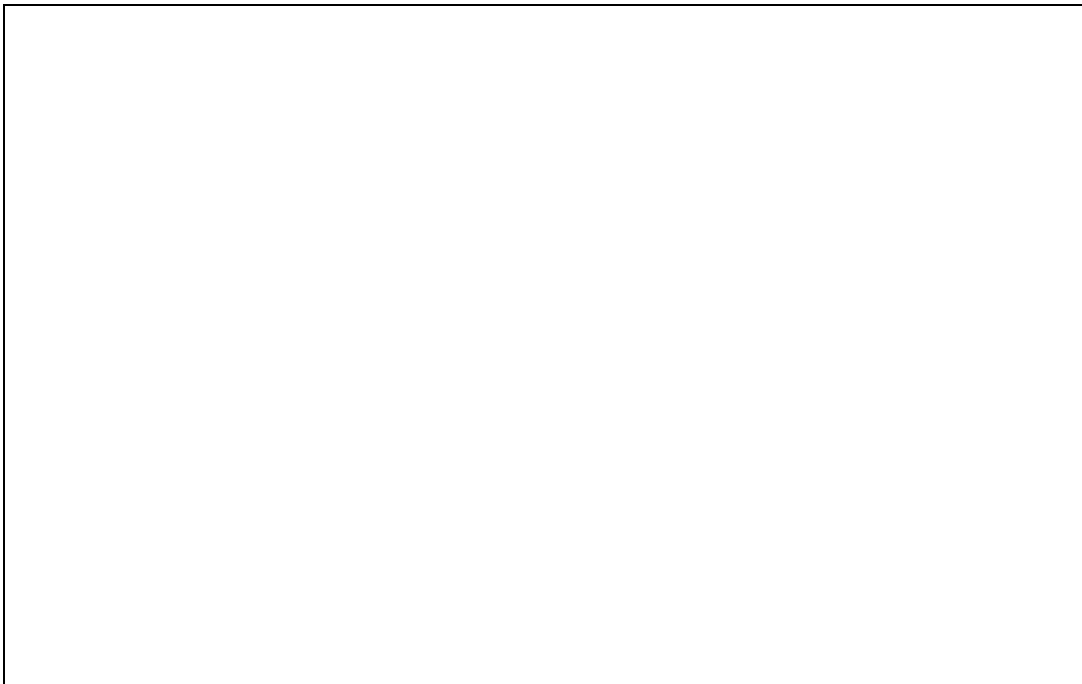
3. (20 points) A diffusion resistor is fabricated by doping Si with a group three element. The doping has a uniform concentration of  $10^{18}\text{cm}^{-3}$ . The thickness of the diffusion region is unknown but the measured resistance of a long rectangular resistor with  $L = 100\mu$  and  $W = 5\mu$  is  $3\text{k}\Omega$ . (a) Estimate the free carrier electron and hole densities.



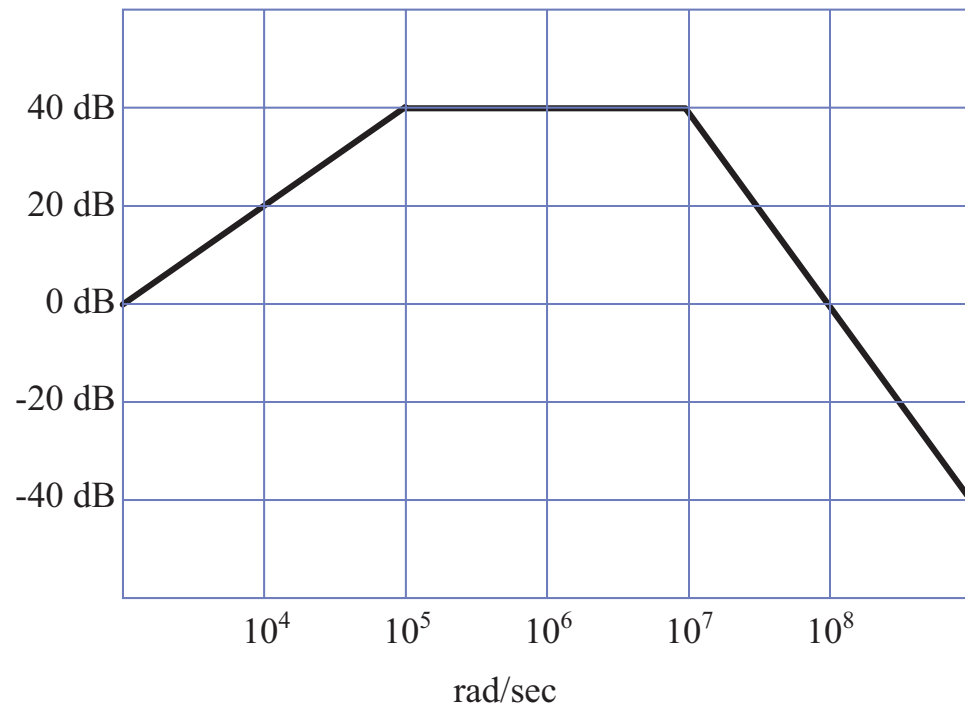
(b) Calculate the thickness of the diffusion region (be careful with units of cm!).



(c) The same sample is now heated to extend the diffusion region. It is known that the diffusion region grows uniformly by  $5\mu$ , so the device now has a length of  $L = 110\mu$ , a width  $W = 15\mu$ , and a thickness that increases by  $5\mu$ . Calculate the resistance of the diffusion resistor.

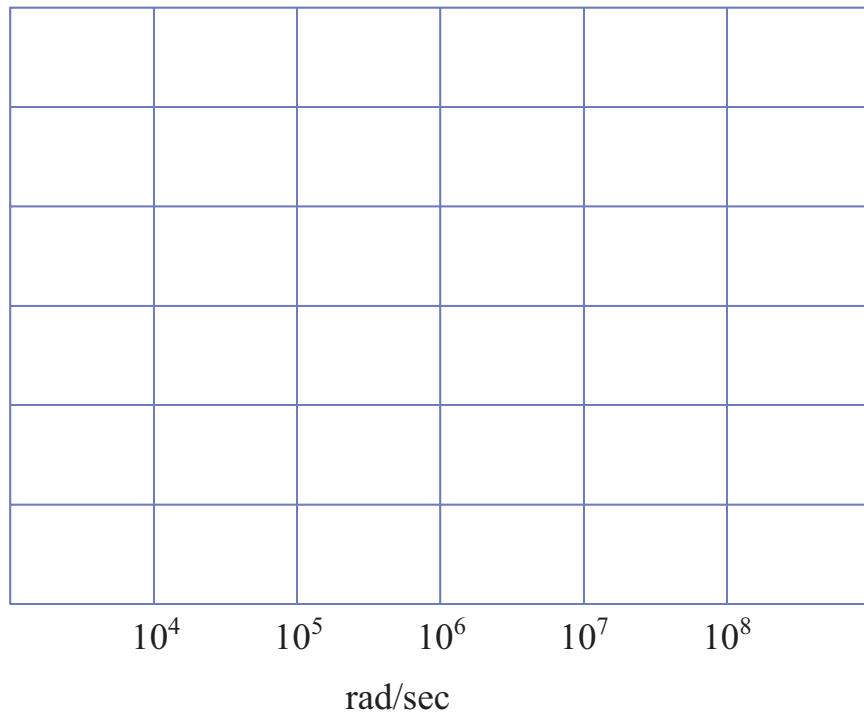


4. (20 points) Given the Bode plot shown below

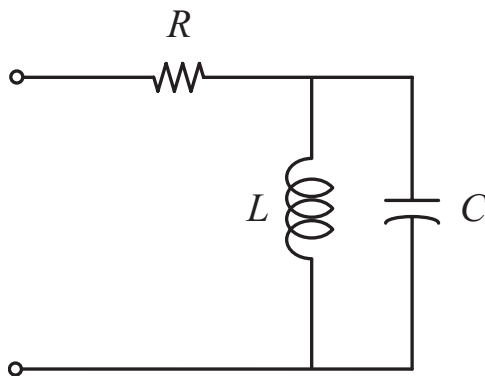


(a) write the transfer function.

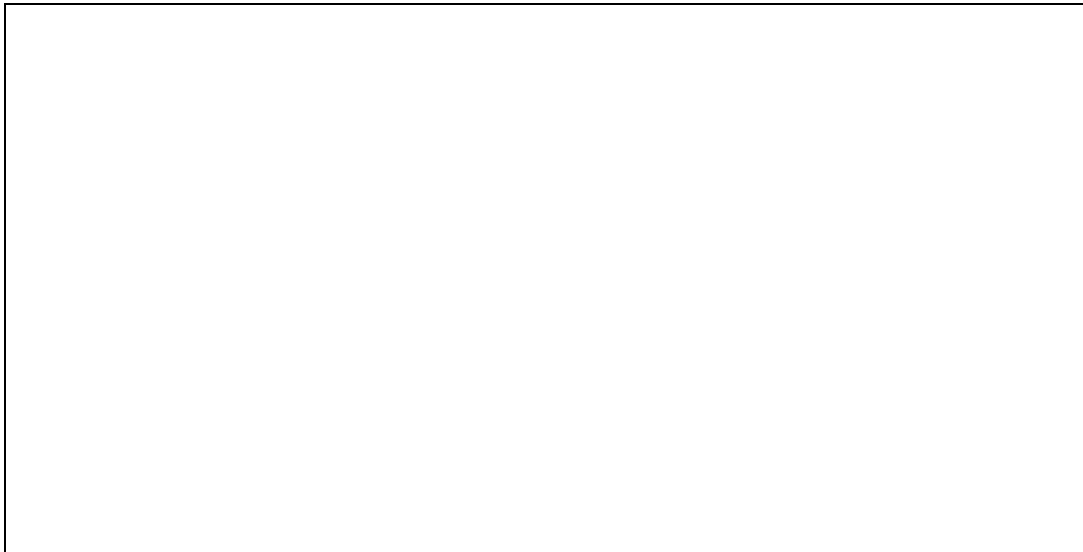
(b) Draw the phase response assuming all poles and zero occur in the left half plane (in the provided graph).



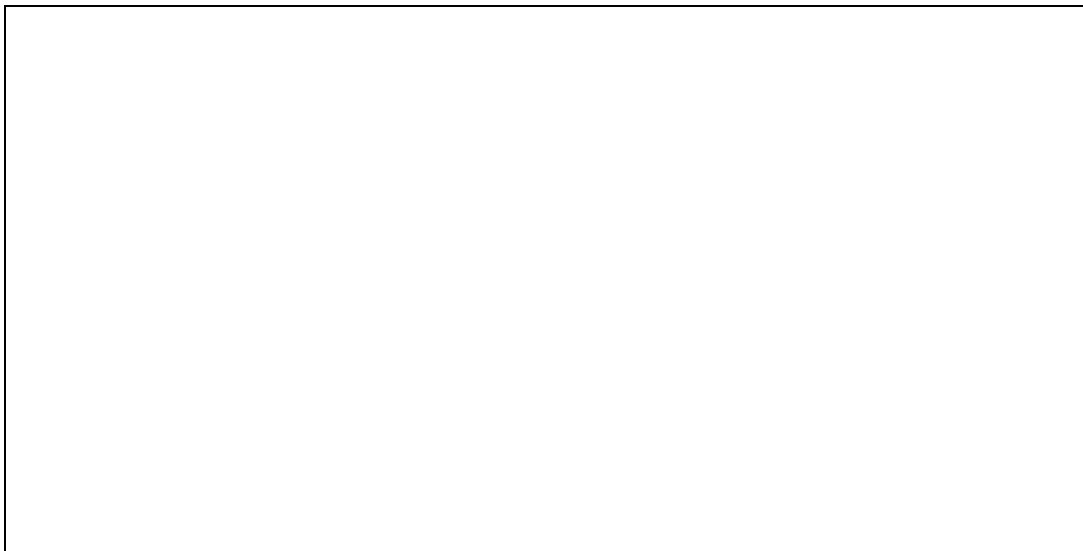
5. (20 points) Given the circuit shown below



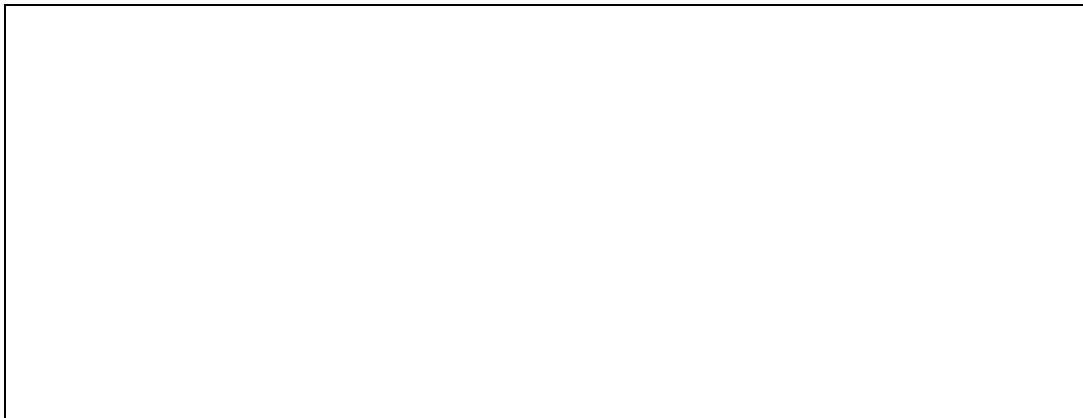
(a) Calculate the impedance transfer function. Identify the poles and zeros assuming that  $\omega_0 L/R = 10$ .



(b) The denominator for the transfer function is a second order function. What is the Q of the denominator of the transfer function?



(c) What happens at frequency  $\omega = \sqrt{LC}$ ? Explain qualitatively the operation of the circuit at this frequency.



## Useful Equations:

An electron has  $1.602C$  of charge.

A complex number  $z = x + jy$  can be written in polar coordinates as  $z = re^{j\theta}$  where  $r^2 = x^2 + y^2$  and  $\tan(\theta) = y/x$ .

The magnitude of a complex ratio  $z = \frac{v}{w}$  can be simplified by taking the magnitude of the numerator divided by the denominator:  $|z| = \frac{|v|}{|w|}$ . The phase is the difference of the phase of the numerator and the denominator:  $\angle z = \angle v - \angle w$ .

The impedance transfer function is defined as the ratio of the voltage to the current:  $Z = V/I$ . Similarly, the admittance is  $Y = I/V$ .

The average power dissipated can be computed in terms of phasors:

$$P = \operatorname{Re}\left(\frac{I^*V}{2}\right) = \operatorname{Re}\left(\frac{V^*I}{2}\right)$$

A second order system can be written in the following canonical form:

$$\left(j\frac{\omega}{\omega_0}\right)^2 + \left(j\frac{\omega}{\omega_0}\right)2\zeta + 1$$

where  $\omega_0$  is the natural frequency (also called the natural resonant frequency) and  $\zeta$  is the damping factor.

The electrical conductivity of a metal is given by the following expression

$$\sigma = q^2 \left( \frac{N^+ \tau^+}{M^+} + \frac{N^- \tau^-}{M^-} \right)$$

where  $q$  is the charge of the electrical carriers,  $N$  is the number density of free carriers,  $\tau$  is the average time between collisions, or the mean free time, and  $M$  is the effective mass of the free carriers.

Law of mass action says that at thermal equilibrium, the concentrations of electrons and holes satisfy the following equality

$$np = n_i^2$$

The mobility is defined as the proportionality constant between an applied external electric field and the drift velocity of carriers

$$v_{dr} = \mu E_{ext}$$

The electrical conductivity is related to the mobility by the following equation

$$\sigma = qn\mu_n + qp\mu_p$$

where  $n$  and  $p$  are the density of free carriers.



The resistance of a rectangular structure can be calculated by using the sheet resistance

$$R = R_{sh} \frac{L}{W}$$

where the sheet resistance is defined by

$$R_{sh} = \frac{1}{t\sigma}$$

where  $t$  is the thickness of the rectangular structure.

Mobility can be estimated from the following graph:

