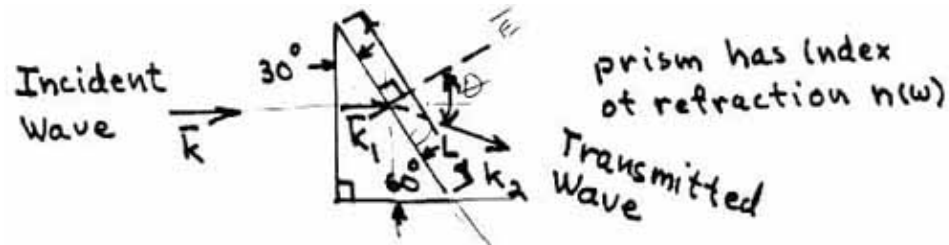


**Electrical Engineering 118**  
**Spring 2001 Midterm 1**  
**Professor Gustafson**

**Problem #1 (30 points)**

A plane wave travels through a right-angle glass prism as shown. The prism materials has an index of refraction which depends upon frequency ( $\omega = 2\pi f$ )

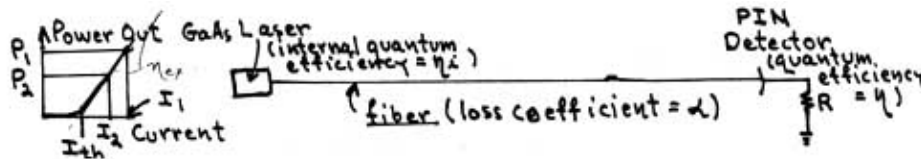


The incident wave has an electric field equal to  $E = E_0 \cos(\omega t - k_x x) = E_0 \cos(\omega t - k \cdot r)$  with  $k = k_x i_x$ , and  $i_x$  a unit vector in the  $x$  direction.  $k$  is known as the  $k$ -vector!

- What is the magnitude of  $k$  in terms of the frequency  $\omega$  and the speed of light  $c$ ?
- What is the magnitude of  $k_1$  for the wave inside the prism?
- What is the magnitude of  $k_2$ , the  $k$ -vector of the wave emerging from the prism (as shown in the figure)
- The field  $E$  is incident perpendicularly on the prism as shown. Use "Snell's law" to obtain the emergence angle  $\theta$  (or its cosine, sine, or tangent) in terms of the index of refraction  $n(\omega)$ ; that is, the value of  $n$  at the frequency  $\omega$ .
- For a small increase,  $\Delta \omega$  in  $\omega$  determine the change  $\Delta \theta$  in angle  $\theta$  due to the change in  $n$  with frequency  $\omega$ , as given by  $(dn/d\omega)(\Delta \omega)$ . Thus obtain an expression for  $(\Delta \theta)/(\Delta \omega)$ , the prism dispersion coefficient, in terms of  $dn/d\omega$  and  $n(\omega)$ .
- What is the "resolution" in this case (the minimum value of  $\Delta \omega$  that can be discerned)? Hint -- this is a diffraction problem! Express your answer in terms of  $(\omega \cdot dn/d\omega)$ ,  $c$ , the speed of light, and  $L$  (the hypotenuse of the prism).
- A typical value of  $\omega \cdot dn/d\omega$  is 0.875 (flint glass at red wavelengths). What length must the prism hypotenuse be to resolve 100GHz?

**Problem #2 (30 points)**

An optical communication system has the components with corresponding characteristics shown.



- Express the detector current  $I_{d_1}$  and  $I_{d_2}$  obtained if the laser transmitter is driven respectively with currents  $I_1$  and  $I_2$ . Express these in terms of  $P_1$  (the optical power obtained by exciting the laser with current  $I_1$ ) and  $P_2$ ,  $\alpha$ ,  $L$ , where  $\alpha$  is the loss coefficient of the optical fiber and  $L$  is its length;  $\eta_d$ , the quantum efficiency of the detector; the frequency  $f$ ; and the electronic charge  $e$ .
- Express  $I_{d_1}$  and  $I_{d_2}$  in terms of  $I_1$  and  $I_2$  as well as  $\eta_i$ , the internal quantum efficiency of the laser diode,  $\alpha_i$ , the internal loss of the laser diode, and  $\alpha_m$ , the mirror loss ( $\text{cm}^{-1}$ ) coefficient of the laser diode.
- Assuming that the detection is thermal noise limited with resistance,  $R$ , what is the signal to noise ratio defined as  $K^2 = (\Delta I_d)^2 / (i_{NT})^2$ , where  $\Delta I_d = I_{d_1} - I_{d_2}$ . In terms of  $T$ , the temperature,  $R$ , the resistance,  $\Delta f$ , the bandwidth and  $I_1$  and  $I_2$  and the parameters which define them.
- If an APD is used rather than a PIN diode, and  $M$  is the current amplification and  $F$  the noise enhancement factor, what is the minimum value of  $M$  required to guarantee that the detection is shot-noise limited? Assume that  $F$  is approximately constant.

**Problem #3 (10 points)**

The system in problem 2 is used as a digital transmission system with  $I_2$  corresponding to a 1 and  $I_1$  corresponding to a 0. The signal to be sent digitally is sampled at a 100Mhz rate and the laser can be switched so that 10 bits per sample period can be obtained. How many levels can be used to quantize the signal before the noise makes it impossible to discern one level from another? What is the corresponding signal to noise ratio for this encoding process?

**Solutions!**

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