

**EECS 40, Fall 2009
Midterm Exam #1**

Oct 1, 2009
Total Time Allotted: 80 minutes
Total Points: 100

***DO ALL WORK ON EXAM PAGES
(total ? pages)***

1. This is a closed book exam. You are allowed to bring one sheet (8.5" x 11") of notes.
2. You can use a calculator. NO cell phone or computer.
3. If you put down the wrong answer, partial credits will be given only if you show the correct steps.
4. Points will be taken off for answers without units.

Last (Family) Name: _____ **SOLUTIONS** _____

First Name: _____

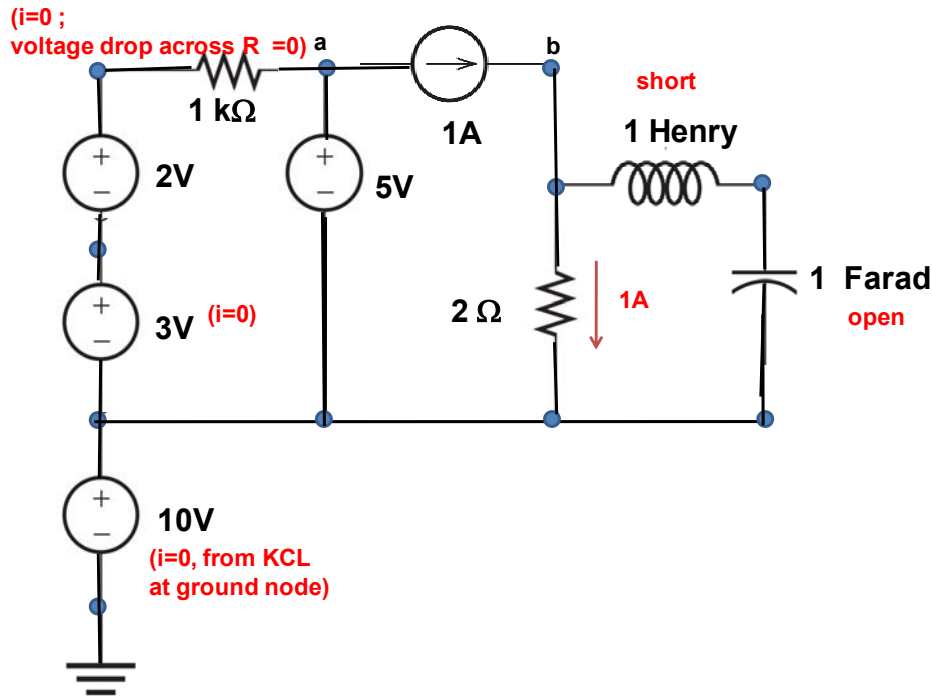
Student ID: _____ Discussion Session (# or TA): _____

Signature: _____

Problem 1 (20 points)	
Problem 2 (20 Points)	
Problem 3 (10 Points)	
Problem 4 (20 Points)	
Problem 5 (15 Points)	
Problem 6 (15Points)	
TOTAL (100 points)	

Problem 1 Circuit elements, power, and energy (20 points total)

The following circuit is at steady state condition (i.e. turned on for a long time)



(a) (2 points each) Check the appropriate box for power supplied/ absorbed by the following circuit elements

	Supplies power	Absorbs power	Zero power
10V voltage source			✓
3V voltage source			✓
1kΩ resistor			✓
5V voltage source	✓		
1A current source		✓	
1 Farad capacitor			✓
1 Henry inductor			✓

(2 points) Calculate the power dissipated by the 2Ω resistor

Capacitor acts like open, Current through 2Ω resistor = 1A .
 Power = $i^2 R = 2 \text{ Watts}$

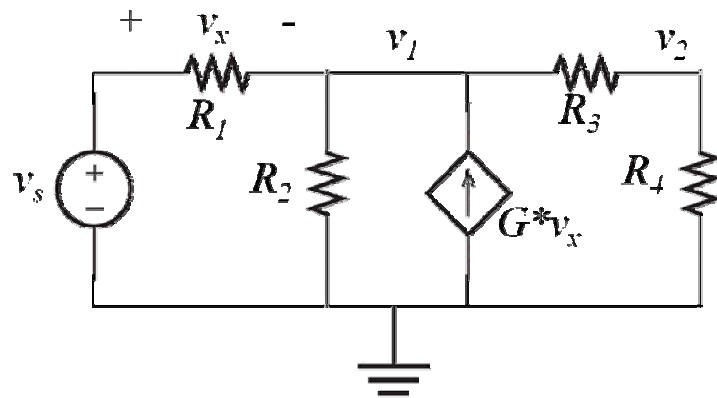
(2 points) Calculate the voltage (v_{ab}) across the 1A source.

$$v_{ab} = +5V - 1A \cdot 2\Omega = +3V$$

(2 points) Calculate the energy stored in the 1F capacitor.

Voltage across capacitor = voltage across resistor = $1A \cdot 2\Omega = 2V$
 Energy = $\frac{1}{2} (CV^2) = \frac{1}{2} (1 \cdot 2^2) = 2 \text{ Joules}$

Problem 2 Nodal Analysis (20 points)



Let $R_1 = 1 \text{ k}\Omega$, $R_2 = 2 \text{ k}\Omega$, $R_3 = 3 \text{ k}\Omega$, $R_4 = 4 \text{ k}\Omega$, $G = 5 \text{ mS}$, $v_s = 2.5 \text{ V}$.

- a) Set up the equations to solve this problem by nodal analysis (analytically – no numbers). Eliminate current variables and express all of your equations in terms of voltages only. (10 pts)

We have four nodes, of which one is reference and one is fixed at v_s . This leaves two unknown node voltages, which require two equations. Summing the currents into each of the other two nodes, and re-expressing currents in terms of voltages using Ohm's law, yields the following equations:

$$\frac{v_s - v_1}{R_1} - \frac{v_1}{R_2} + G v_x - \frac{v_1}{R_3 + R_4} = 0$$

$$\frac{v_2 - v_1}{R_3} + \frac{v_2}{R_4} = 0$$

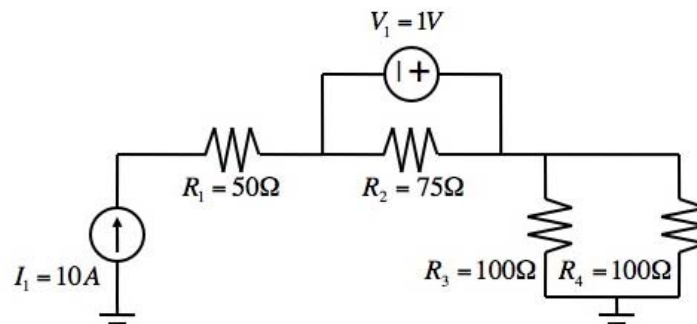
Modifications of these equations (such as re-expressing v_x in terms of v_1 and v_2) are also accepted.

- a) Solve the equations. Give numerical answers to 3 significant digits. (10 pts)

$$v_1 = v_s \frac{\frac{1}{R_1} + G}{\frac{1}{R_1} + G + \frac{1}{R_2} + \frac{1}{R_3 + R_4}} = 2.26 \text{ V}$$

$$v_2 = v_1 \frac{R_4}{R_3 + R_4} = 1.29 \text{ V}$$

Problem 3 Superposition (10 points)



For the circuit above, mark all currents you will use to analyse the circuit on the figure. Determine the current values flowing through R_2 , R_3 and R_4

[Hints: use superposition]

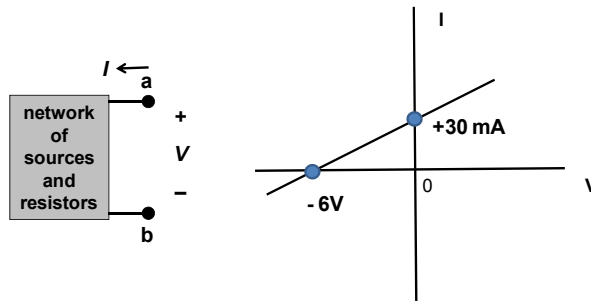
Apply I_1 first-Since R_1 is in series with the current source, it does not impact the circuit. Also, R_2 is shorted by the voltage source, so no current flows through it. Current from I_1 splits equally between R_3 and R_4 , so each carries 5A.

Apply now V_1 . Since I_1 is open, no current flows through R_3 or R_4 . The 1V source generates current that recirculates through R_2 , so $I[R_2]=13.3\text{mA}$

So $I[R_2]=13.3\text{mA}$, $I[R_3]=I[R_4]=5\text{ A}$

Problem 4 Thevenin and Norton Equivalent (20 points)

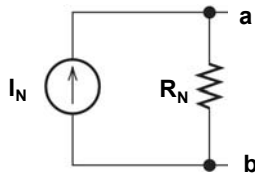
Looking into terminals a and b of a linear circuit, the I-V characteristic is plotted below.



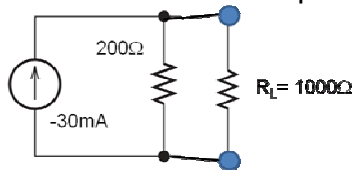
(a) (6 points) Calculate values of the Norton Current I_N and the Norton Equivalent resistance R_N .

$$I_N = -i_{sc} = -30\text{mA},$$

$$R_N = -v_{oc}/i_{sc} = -6\text{V}/-30\text{mA} = 200\ \Omega$$

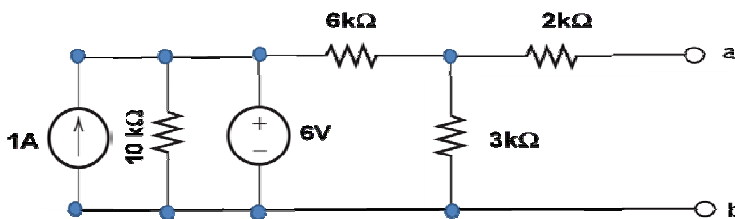


(b) (4 points) A $1\text{k}\Omega$ load resistor is connected to the Norton Equivalent Circuit. Calculate the power dissipated by the load resistor

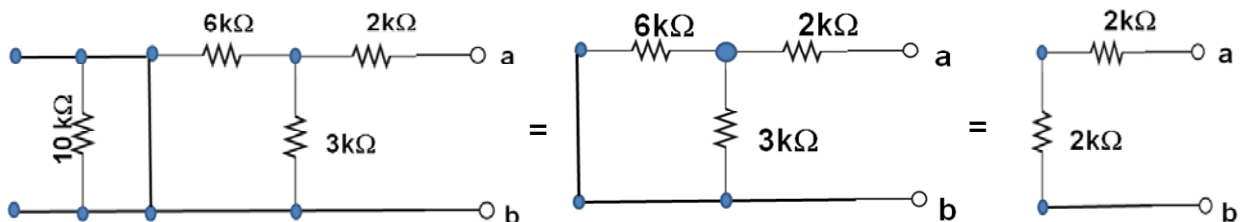


Using current divider formula, current through load resistor
 $= -30\text{mA} [200/(200+1000)] = -5\text{mA}$
 Power = $I^2 R = (5 \cdot 10^{-3})^2 \cdot 1000 = 25\ \text{mWatt}$

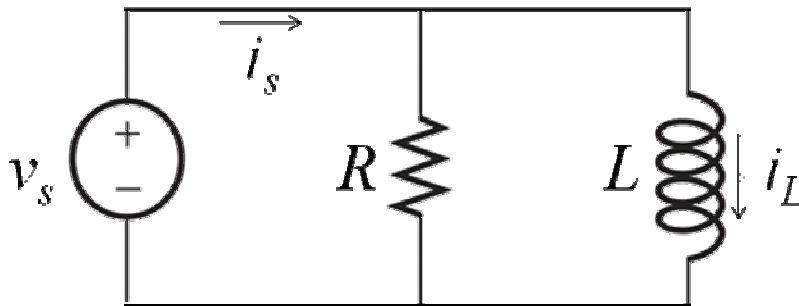
(c) (10 points) Find the Thevenin Equivalent of the following circuit, looking into terminals a and b:



With a and b open circuited, $V_{Th} = v_{oc} = \text{voltage across } 3\text{k}\Omega \text{ resistor} = 6\text{V} [3/(3+6)] = +2\text{V}$
 By setting all independent sources to zero, we get by inspection $R_{Th} = 4\text{k}\Omega$



Problem 5 RL Circuit (15 points)



$R = 3 \Omega$, $L = 1 \text{ H}$, $v_s = 2 \text{ V}$
Initial condition: $i_L = -1.5 \text{ A}$

Plot the currents i_L and i_s and the stored energy W on the axes given. Show the numerical values at $t = 0$ and $t = 1 \text{ s}$. (5 pts each)

i_L :

The voltage across the inductor is known to be v_s at all times, so we can immediately integrate the voltage to find the current. $L=1\text{H}$ in this problem.

$$i_L = \int v_s dt = v_s \cdot t + i_L(0)$$

The constant of integration is simply our initial condition $i_L(0) = -1.5 \text{ A}$.

Evaluate at $t = 1 \text{ s}$ to find $i_L(1 \text{ s}) = 2 + (-1.5) = 0.5 \text{ A}$.

Note also that this curve crosses 0 at $t = 0.75 \text{ s}$.

i_s :

We now apply KCL to find:

$$i_s = i_L + \frac{v_s}{R}$$

The curve for i_s is the same as the curve for i_L , but shifted up by $v_s/R = 2/3 \text{ A}$. In particular, $i_s(0) = -0.83 \text{ A}$, $i_s(1 \text{ s}) = 1.17 \text{ A}$.

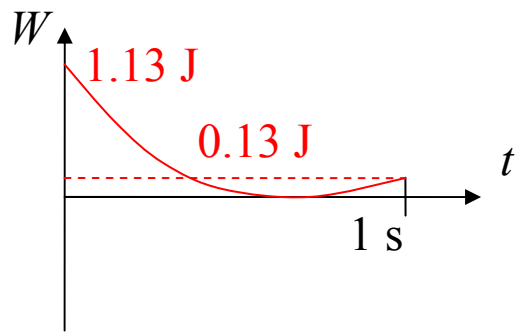
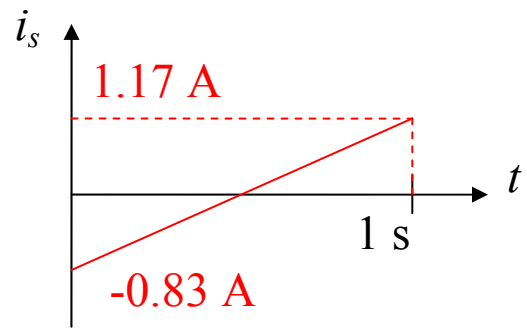
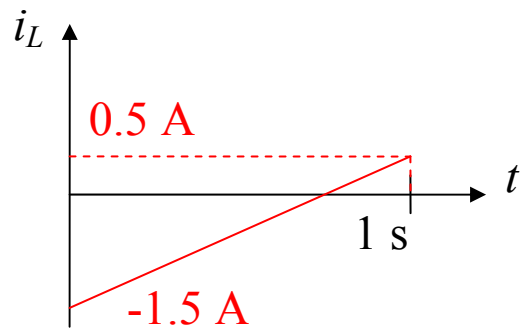
W :

Apply the formula for energy stored in an inductor:

$$W = \frac{1}{2} L \cdot i_L^2$$

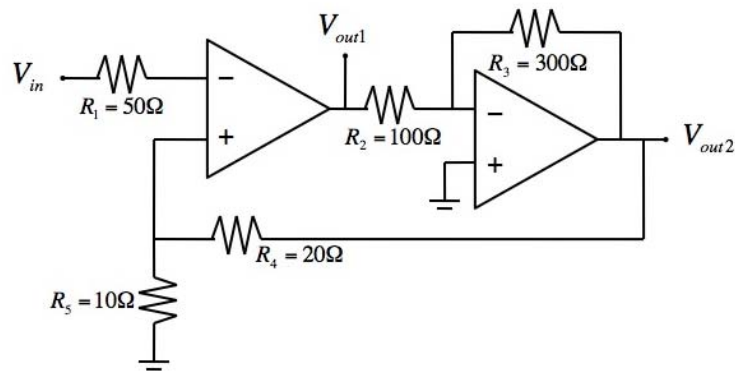
Since i_L is linear, W is a parabola. Its center coincides with the zero-crossing of i_L . The two other values of interest are $W(0) = 1.13 \text{ J}$, $W(1 \text{ s}) = 0.13 \text{ J}$.

Note that stored energy cannot be negative.



Problem 7 Op Amp (15 points)

For the op-amp circuit below



a) Discuss briefly the type of feedback for this OP Amp circuit (5 pts)

Suppose the positive terminal of the first operational amplifier is increased- this will increase its output. Since the second op-amp is configured as an inverting amplifier, the output of the second op-amp will then decrease, and thanks to the resistive divider R_3 - R_4 , so will the positive terminal of the op-amp. Hence, the feedback is negative.

b) Solve for the output voltages v_{o1} and v_{o2} in the terms of the input (10 pts)

Since feedback is negative, virtual short circuit can be applied.

From the inverting configuration, we find $V_{out2} = -3V_{out1}$.

The positive terminal of the op-amp must be equal to V_{in} , but also to the resistive divider between R_3 and R_4 , i.e. $V_{out2}/3$.

Hence, $V_{out2}/3 = -V_{out1} = V_{in}$,

or $V_{out1} = -V_{in}$; $V_{out2} = 3V_{in}$.